Problems on logic

1. For each of the following, determine the truth value of the given statement when $p$ and $q$ are true and $r$ is false.
   
   (a) $\sim(p \Rightarrow q)$
   (b) $\sim p \Rightarrow r$
   (c) $p \Rightarrow (r \Rightarrow q)$

2. Suppose $p \Rightarrow q$ is false. Determine the truth value of $\sim (p \land q) \Rightarrow q$.

3. For each of the following, choose specific statements for $p$, $q$, and $r$ so that the given statement is true. Also choose a different set of specific statements for $p$, $q$, and $r$ so that the given statement is false.
   
   (a) $(\sim (p \land q)) \Rightarrow r$
   (b) $r \Rightarrow (p \lor q)$

4. For each of the following, construct a truth table to show that the given statements are logically equivalent.
   
   (a) $\sim (p \lor q) \leftrightarrow (\sim p) \land (\sim q)$
   (b) $p \Rightarrow q \leftrightarrow (p \land (\sim q))$
   (c) $p \lor q \leftrightarrow (\sim p) \Rightarrow q$

5. A tautology is a statement that is true under all conditions. Construct a truth table to show that $p \lor (\sim p)$ is a tautology.

6. Construct a truth table to show that

$$[(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

is a tautology. Note that we did part of this in class but failed to consider all possible combinations of truth values for $p$, $q$, and $r$. You will need 8 rows in your truth table.

7. A contradiction is a statement that is false under all conditions. Construct a truth table to show that $p \land (\sim p)$ is a contradiction.