Practice with judicious guessing

In the problems below, you will work with the differential equation

\[ \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = f(t) \]

for various choices of the nonhomogeneous term \( f(t) \).

1. Solve the related homogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 0 \).

2. Solve the nonhomogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 6 \). Hint: Try \( y_p = A \).

3. Solve the nonhomogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 3t \). Hint: Try \( y_p = A + Bt \).

4. Solve the nonhomogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = -2e^{2t} \). Hint: Try \( y_p = Ae^{2t} \).

5. Solve the nonhomogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = e^{4t} \). Hint: First try \( y_p = Ae^{4t} \). When this doesn’t work, articulate what is going on here. Then try \( y_p = At e^{4t} \).

6. Solve the nonhomogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 6 \cos(3t) \). Come up with your own judicious guess.

7. Solve the nonhomogeneous problem \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 6e^{3it} \). Come up with your own judicious guess.

8. From your particular solution in Problem 7, extract a particular solution for Problem 6.

9. From your particular solution in Problem 7, extract a particular solution for \( \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 6 \sin(3t) \).