Problems: Total from volume density

1. Relative to a chosen cartesian coordinate system, a solid object sits in the first octant bounded by $z = 4 - x^2 - y$ and the coordinate planes. The object has a non-uniform composition so that the volume mass density is given by $\delta(x, y, z) = 3z$. Compute the total mass of the solid.

   Answer: $M = \frac{1024}{35}$

2. Charge is distributed throughout a rectangular region of space having dimension $L$ by $W$ by $H$ so that the volume charge density is proportional to the square of the distance from one corner, reaching a maximum of $\delta_0$ at the far corner. Compute the total charge $Q$.

   Answer: $Q = \frac{1}{3} LWH\delta_0$

3. A solid (right circular) cylinder of radius $R$ and height $H$ has a non-uniform composition so that the volume mass density is proportional to the distance from the lateral surface reaching a maximum $\delta_0$ along the central axis. Compute the total mass $M$.

   Answer: $M = \frac{1}{3} \pi R^2 H \delta_0$

4. Charge is distributed throughout a solid (right circular) cone of radius $R$ and height $H$ so that the volume charge density is proportional to the square of the distance from the vertex of the cone reaching a maximum of $\delta_0$ along the edge of the base of the cone. Compute the total charge $Q$.

   Answer: $Q = \frac{1}{10} \pi R^2 H \frac{R^2 + 2H^2}{R^2 + H^2} \delta_0$

5. A solid sphere of radius $R$ has a non-uniform composition so that the volume mass density is proportional to the distance from the center of the sphere reaching a maximum of $\delta_0$ along the surface. Compute the total mass $M$. Compare this mass to the total mass for a solid sphere of the same radius having uniform composition with mass density $\delta_0$.

   Answer: $M = \pi R^3 \delta_0$

6. A solid sphere of radius $R$ has a non-uniform composition so that the volume mass density is proportional to the distance from the surface of the sphere reaching a maximum of $\rho_0$ at the center. Compute the total mass $M$. Compare this total mass to the total mass for a solid sphere of the same radius having uniform composition with mass density $\delta_0$. Also, compare this total mass to the total mass for the sphere in Problem 5.