Problem: Charge is distributed on a disk of radius $R$ so that the area charge density is proportional to the distance from the center, reaching a maximum value $\sigma_0$ at the far edge. Compute the total charge.
Total from area density

\[ Q = \int\int_{\text{disk}} \sigma \, dA \]

\[ = \int_{a}^{b} \int_{c}^{d} \sigma(x, y) \, dy \, dx \]

\[ = \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \sigma(x, y) \, dy \, dx \]

\[ = \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{\sigma_0}{R} \sqrt{x^2 + y^2} \, dy \, dx \]

\[ = \text{not fun to evaluate} \]

Turn to a different coordinate system: polar coordinates.