1. Find the 101st derivative of $z^2e^{-5z^3}$ for $z = 0$ (without using computing technology). (15 points)

2. Compute the Taylor series for $\log z$ based at $z_0 = 1$ and give the region of convergence for this series. (15 points)

3. For each of the following regions, find the Taylor or Laurent series expansion for the function $f(z) = \frac{1}{1+z^2} + \frac{1}{3-z}$ that is valid for that region. (10 points each)
   - (a) $|z| < 1$
   - (b) $1 < |z| < 3$
   - (c) $3 < |z|$

4. We have previously determined the Laurent series for the function $f(z) = \frac{1}{4z-z^2}$ that converges for $0 < |z| < 4$. We did this by rewriting the function so that we could take advantage of the geometric series result. As usual, let $b_1$ be the coefficient on the $1/z$ term. Compute $b_1$ directly and compare this with the value of $b_1$ we got in our previous work. (15 points)

5. Your friend makes the following argument:

   Consider the function defined by
   
   $$f(z) = \cdots + \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{z} + 1 + z + z^2 + z^3 + \cdots$$

   Note that
   
   $$1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots = \frac{1}{1 - 1/z} = \frac{z}{1-z}.$$ 

   Also note that
   
   $$z + z^2 + z^3 + \cdots = z(1 + z + z^2 + \cdots) = z \frac{1}{1-z} = \frac{z}{1-z}.$$ 

   Therefore,
   
   $$f(z) = -\frac{z}{1-z} + \frac{z}{1-z} = 0.$$ 

   In other words, $f$ is the zero function.

   Is your friend’s argument correct? If not, explain any flaws in the argument. Is the conclusion correct? If $f$ is not the zero function, determine what is true about $f$ as a function. (15 points)

6. Prove the following: If $f$ is entire and $\lim_{z \to \infty} \frac{f(z)}{z} = 0$, then $f$ is constant. (10 points)