Computing probabilities

**Example:** For the process of summing the values on a pair of fair dice, the sample space is \( S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \). We have worked out the following probability distribution for this process:

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
</table>

1. What is the probability of getting 10 or more on one roll?

   Event: \( A = \{10, 11, 12\} \)

   Probability: \( P(A) = \)

2. What is the probability of getting less than 10 on one roll? Compute this probability directly from the probability distribution.

   Event: \( B = \{2, 3, 4, 5, 6, 7, 8, 9\} \)

   Probability: \( P(B) = \)

3. How is event \( B \) related to the sample space \( S \) and event \( A \)?

4. How is the probability \( P(B) \) related to the probabilities \( P(S) \) and \( P(A) \)?

5. Use the idea from (4) and your result from (1) to compute the probability of getting less than 10 on one roll. Compare with your result from (2).
Example: For the process of a fair coin, the sample space is $S = \{H, T\}$. We assume the following probability distribution for this process:

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

(a) What is the probability of getting heads on one flip of a fair coin?

(b) What is the probability of getting heads on a second flip of a fair coin?

(c) What is the probability of getting heads on both of two successive flips of a fair coin? You can think of this as the event “heads on the first flip AND heads on the second flip”.

(d) Let $A$ be the event “heads on the first flip”. Let $B$ be the event “heads on the second flip”. How is $P(A$ and $B)$ related to $P(A)$ and $P(B)$? Write this as a formula.

(e) What is the probability of getting 5 heads every time on 5 successive flips of a coin?

(f) If you look at 96 different instances of 5 successive flips of a coin, how many times would you expect to get all heads? Would it surprise you to not get 5 heads in a row in 96 tries?