Problems from Section 2.1

1. Use the Bisection method to find \( p_3 \) for \( f(x) = \sqrt{x} = \cos x \) on \([0, 1]\).
   Note: The text denotes the value of the \( n \)th approximation by \( p_n \).

5. Use the Bisection method to find solutions accurate to with \( 10^{-5} \) for the following problems.
   (a) \( x - 2^{-x} = 0 \) for \( 0 \leq x \leq 1 \)

7. (a) Sketch the graphs of \( y = x \) and \( y = 2 \sin x \).
   (b) Use the Bisection method to find an approximation to within \( 10^{-5} \) to the first positive value of \( x \) with \( x = 2 \sin x \).

11. Let \( f(x) = (x + 2)(x + 1)x(x - 1)^3(x - 2) \). To which zero of \( f \) does the Bisection method converge when applied on the following intervals?
   (a) \([-3, 2.5]\] (b) \([-2.5, 3]\] (c) \([-1.75, 1.5]\] (d) \([-1.5, 1.75]\]
   Note: For each, try to determine the relevant zero with a minimal amount of computation. That is, try to avoid a “brute force” approach such as iterating the bisection method 1000 times and then checking which zero the resulting approximation is near.

18. The function \( f(x) = \sin(\pi x) \) has zeros at every integer. Show that when \(-1 < x < 0\) and \(2 < b < 3\), the Bisection method converges to
   (a) 0, if \( a + b < 2 \) (b) 2, if \( a + b > 2 \) (c) 1, if \( a + b = 2 \)

Programming Problem  Modify the implementation of the bisection method from class to include a check that the original interval is valid.