1. For each of the following, compute all values of the given expression (giving results in the form $a + bi$). Show enough detail so it is clear you can do this without the aid of a calculator. (4 points each)

(a) $e^{\sqrt{3} - i}$
(b) $\sin(\sqrt{3} - i)$
(c) $\log(\sqrt{3} - i)$
(d) $\sinh(\sqrt{3} - i)$

2. Let $f(z) = (x^2 + 2y) + i(x^2 + y^2)$ for $z = x + iy$.

(a) At what points is $f$ differentiable? (6 points)
(b) At what points is $f$ analytic? (6 points)

3. Consider the function $f(z) = e^{-z^2}$.

(a) Compute $f'(i)$. (6 points)
(b) Use the result for (a) to describe the geometric effect of the mapping $f$ near the point $z = i$. (6 points)

4. (a) Find the real and imaginary parts of $f(z) = e^{ez}$. (6 points)
(b) Give an argument to show that $f(z) = e^{ez}$ is entire. Hint: Don’t even think about using the Cauchy-Riemann equations. (4 points)

5. Consider the function $u(x, y) = \frac{y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$.

(a) Show that $u(x, y)$ is harmonic. (4 points)
(b) Find a harmonic conjugate $v(x, y)$ for $u(x, y)$. (6 points)
(c) Express the function $f(z) = u(x, y) + iv(x, y)$ in terms of $z$. (4 points)

6. Find all values of $(1 + i)^i$ and plot these values (or at least plot enough to make clear where these are). (12 points)

7. Prove the identity $\cos^2 z + \sin^2 z = 1$. (12 points)

8. (a) Let $z$, $\alpha$, and $\beta$ be complex numbers. Prove the following: If $z^\alpha$, $z^\beta$, and $z^{(\alpha+\beta)}$ are all defined using the same branch of log, then $z^\alpha z^\beta = z^{(\alpha+\beta)}$. (6 points)

(b) Let $z_1$, $z_2$, and $\alpha$ be complex numbers. Prove the following: Branches for $\log(z_1)$, $\log(z_2)$, and $\log(z_1 z_2)$ can be chosen so that $(z_1 z_2)^\alpha = z_1^\alpha z_2^\alpha$. (6 points)