The forced undamped spring

An undamped spring-mass system with an external sinusoidal driving force is modeled by

\[ m x''(t) + k x(t) = E_0 \sin(\beta t). \]

We solved this in class. The particular solution for the nonhomogeneous problem depends on whether \( \beta \neq \omega_0 \) or \( \beta = \omega_0 \) where

\[ \omega_0 = \sqrt{\frac{k}{m}}. \]

1. Find the specific solution for the case \( \beta \neq \omega_0 \) with the initial conditions \( x(0) = x_0 \) and \( x'(0) = v_0 \).

2. Find the specific solution for the case \( \beta = \omega_0 \) with the initial conditions \( x(0) = x_0 \) and \( x'(0) = v_0 \).

3. Show that the specific solution for \( \beta = \omega_0 \) is a limit of the corresponding specific solution for \( \beta \neq \omega_0 \).