1. Prove the identity \( \tan \left( \frac{1}{i} \log \left( \frac{1 + iz}{1 - iz} \right)^{1/2} \right) = z \).

2. Evaluate \( \int_C (z^2 + 1)^{1/2} \, dz \) where \( C \) is the circle of radius \( \frac{1}{2} \) centered at the origin and \( w^{1/2} \) is defined using the branch of the square root function with \( -\pi < \arg w < \pi \).

3. Evaluate \( \int_0^\infty \frac{x^{1/4}}{1 + x^5} \, dx \).

4. Prove any two of the following three claims. Submit attempts for only two.
   
   (a) If a function \( f \) is analytic on a domain \( A \), then the function \( g \) defined by \( g(z) = \left[ f(\bar{z}) \right]^2 \) is analytic on the domain \( \bar{A} = \{ z \mid \bar{z} \in A \} \).

   (b) There is no function \( f \) that has both of the following properties: \( f \) is analytic everywhere except the origin and \( f \) has derivative \( f'(z) = \frac{1}{z} \) for \( z \neq 0 \).

   (c) If \( P(z) \) is a polynomial and \( C \) is a simple closed curve with no roots of \( P(z) \) on \( C \), then
      \[
      \frac{1}{2\pi i} \oint_C \frac{P'(z)}{P(z)} \, dz
      \]
      equals the number of roots of \( P(z) \) inside \( C \) counting multiplicities.

5. Suppose the Math Club publishes a monthly newsletter for math and science majors. Write an article that describes complex analysis for this newsletter. Consider your target audience to be math and science majors who have completed the calculus sequence and linear algebra but who have not taken complex analysis. Your goal is to give those students guidance in making an informed decision about taking a course in complex analysis. Focus on aspects of complex analysis that you think are important or interesting. You do not need to summarize every idea that we have covered this semester.

I will evaluate your article using the criteria described in the rubric on the reverse.