1. Consider the temperature \( T \) as a function of position \((x, y, z)\) in space. Suppose measurements are made to determine the values

\[
T(1, 4, 2) = 27, \quad T_x(1, 4, 2) = 3.2, \quad T_y(1, 4, 2) = -1.3, \quad \text{and} \quad T_z(1, 4, 2) = 2.3.
\]

(a) Construct the linearization of \( T \) based at the point \((1, 4, 2)\). 

(b) Use your linearization from (a) to estimate \( T \) at the point \((1.07, 4.15, 1.98)\).

2. A *Norman window* has the shape of a rectangle surmounted by a semidisc as shown in the figure. With \( W \) as the width of the rectangle and \( H \) as the height of the rectangle, the area of the window is given by

\[
A = WH + \frac{1}{8}\pi W^2.
\]

(a) Compute the linear relation among the differentials \( dA, dW, \) and \( dH \).

(b) Consider the values \( W = 100 \) and \( H = 20 \). For these values, use your relation from (a) to compare the effect on \( A \) of equal-size changes in \( W \) and \( H \).

3. Consider the function \( f(x, y) = x \sin(x + y) \)

(a) Show that \((0, \pi)\) is a critical point of \( f \).

   Hint: You can do this by setting up the relevant equations and then substituting in the given point to show it is a solution.

(b) Use the second-derivative test to classify \((0, \pi)\) as a local minimizer, a local maximizer, or neither.
4. Consider the function \( f(x, y) = xy(2 - y) \). For this function, the only critical points are \((0, 0)\) and \((0, 2)\). Find the global minimum and global maximum of this function for the rectangle \(-1 \leq x \leq 4, -1 \leq y \leq 2\). (15 points)

5. Do either one of the following two problems. Circle the number of the problem you submit. (16 points)

(A) In your job as a movie theater manager, you want to determine optimal prices for child tickets and adult tickets. Let \( p_1 \) be the price of a child ticket and \( p_2 \) be the price of an adult ticket (both measured in dollars). Also, let \( q_1 \) and \( q_2 \) be the quantity sold for each ticket type. You understand that the quantity sold will depend on the price. Based on some data, you decide to model the relationships between price and quantity sold using

\[
q_1 = \frac{1000}{p_1^2 p_2} \quad \text{and} \quad q_2 = \frac{2500}{p_2^2}.
\]

Your operating costs are $3 each child ticket sold and $4 for each adult ticket sold. Find the prices that maximize your profits.

Hint: Profit is total revenue minus total cost.

(B) Consider a (right circular) cylinder segment. Use the method of Lagrange multipliers to find the dimensions that give the maximum volume for a given surface area (including top and bottom).

6. Compute the value of the double integral \( \iint_D f \, dA \) where \( f(x, y) = 2xy + x^2 \) and \( D \) is the region in the second quadrant of the \( xy \)-plane bounded by the curves \( x = 0, y = 0, \) and \( y = 9 - x^2 \). As a small part of this, point out where you use Fubini’s Theorem.

Note: You can stop when only arithmetic remains. (15 points)

7. Consider a thin rectangular plate of dimensions \( L \) by \( W \). The materials composing the plate vary from point to point so that the area mass density is proportional to the square of the distance from the center of the plate, reaching a maximum of \( \sigma_0 \) at each of the four corners. Set up an iterated integral to compute the total mass of the plate.

Note: You do not need to evaluate the iterated integral you set up. (12 points)