**Problem:** Design a fence to enclose a rectangular region of area 1200 m\(^2\). Material for one edge (facing the street) costs $50 per meter while material for the other three edges costs $30 per meter.
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**Objective:** Minimize $C = 50l + 30w + 2 \cdot 30w = 80l + 60w$.
**Problem:** Design a fence to enclose a rectangular region of area 1200 m$^2$. Material for one edge (facing the street) costs $50 per meter while material for the other three edges costs $30 per meter.

**Objective:** Minimize $C = 50\ell + 30w + 2 \cdot 30w = 80\ell + 60w$.

**Constraint:** Need $\ell w = 1200$. 
Method 1

Idea:
Solve constraint for one of the variables and then substitute into the objective function to reduce the number of variables.

\[ \ell = 1200 \]
\[ w \]

Therefore

\[ C = 80\ell + 60w = 80(1200) + 60w = 96000 + 60w. \]

Compute

\[ C' = -96000w^2 + 60. \]

Solve

\[ -96000w^2 + 60 = 0 \]

To get

\[ w = \pm 40. \]

Use

\[ w = 40 \]

To get

\[ \ell = \frac{1200}{40} = 30. \]

So build fence with expensive edge of length 30 meters and other dimension of 40 meters.
Method 1

Idea: Solve constraint for one of the variables and then substitute into the objective function to reduce the number of variables.
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\[ \ell = \frac{1200}{w} \]
Method 1

*Idea:* Solve constraint for one of the variables and then substitute into the objective function to reduce the number of variables.

\[ \ell = \frac{1200}{w} \quad \text{so} \quad C = 80\ell + 60w = 80\frac{1200}{w} + 60w = \frac{96000}{w} + 60w. \]
Idea: Solve constraint for one of the variables and then substitute into the objective function to reduce the number of variables.

\[ \ell = \frac{1200}{w} \] so \[ C = 80\ell + 60w = 80\frac{1200}{w} + 60w = \frac{96000}{w} + 60w. \]

Compute \( C' = -\frac{96000}{w^2} + 60. \)
**Method 1**

*Idea:* Solve constraint for one of the variables and then substitute into the objective function to reduce the number of variables.

\[ l = \frac{1200}{w} \quad \text{so} \quad C = 80l + 60w = 80 \frac{1200}{w} + 60w = \frac{96000}{w} + 60w. \]

Compute \( C' = -\frac{96000}{w^2} + 60. \)

Solve \(-\frac{96000}{w^2} + 60 = 0\) to get \( w = \pm 40. \)
Method 1

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Use \( w = 40 \) to get \( \ell = \frac{1200}{40} = 30 \)
**Method 1**

*Idea:* Solve constraint for one of the variables and then substitute into the objective function to reduce the number of variables.

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l = \frac{1200}{w} \quad \text{so} \quad C = 80l + 60w = 80\frac{1200}{w} + 60w = \frac{96000}{w} + 60w.
\]

Compute \( C' = -\frac{96000}{w^2} + 60 \).

Solve \( -\frac{96000}{w^2} + 60 = 0 \) to get \( w = \pm 40 \).

Use \( w = 40 \) to get \( l = \frac{1200}{40} = 30 \).

So build fence with expensive edge of length 30 meters and other dimension of 40 meters.
**Constraint curve** \( A = \ell w = 1200 \)

Level curves for objective \( C = 80\ell + 60w \)

Gradient vectors for constraint \( A = \ell w \)

Gradient vectors for objective \( C = 80\ell + 60w \)
Idea for Method 2

Constraint curve $A = lw = 1200$

**Level curves for objective** $C = 80l + 60w$

Gradient vectors for constraint $A = lw$

Gradient vectors for objective $C = 80l + 60w$
Idea for Method 2

Constraint curve $A = \ell w = 1200$
Level curves for objective $C = 80\ell + 60w$

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Constraint curve $A = \ell w = 1200$
Level curves for objective $C = 80\ell + 60w$
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Gradient vectors for objective $C = 80\ell + 60w$
Idea for Method 2

Maximum or minimum of objective along constraint curve will be at a point where

\[ \vec{\nabla} C = \lambda \vec{\nabla} A \]

for some constant \( \lambda \).
Idea for Method 2

Maximum or minimum of objective along constraint curve will be at a point where

objective level curve is tangent to constraint curve
Idea for Method 2

Maximum or minimum of objective along constraint curve will be at a point where

objective level curve is tangent to constraint curve

\[ \vec{\nabla}C = \lambda \vec{\nabla}A \]

for some constant \( \lambda \)

\[ \vec{\nabla}(C + \lambda A) = 0 \]

for some constant \( \lambda \)
Idea for Method 2

Maximum or minimum of objective along constraint curve will be at a point where

- objective level curve is tangent to constraint curve
- objective gradient $\vec{\nabla} C$ is aligned with constraint gradient $\vec{\nabla} A$
- $\vec{\nabla} C = \lambda \vec{\nabla} A$ for some constant $\lambda$
Idea for Method 2

Maximum or minimum of objective along constraint curve will be at a point where

objective level curve is tangent to constraint curve

$\vec{\nabla}C$ is aligned with constraint gradient $\vec{\nabla}A$

$\vec{\nabla}C = \lambda \vec{\nabla}A$ for some constant $\lambda$

$\vec{\nabla}(C + \lambda A) = \vec{0}$ for some constant $\lambda$