Notes on $L^2(a, b)$

- $L^2(a, b)$ is a vector space
- elements in $L^2(a, b)$ are functions that are square-integrable for the domain $(a, b)$
  - a function is square-integrable if the square of the function is integrable
  - that is, a function is square-integrable if $\int_a^b (f(x))^2 \, dx$ exists
- addition is pointwise addition: $(f + g)(x) = f(x) + g(x)$ for each $x$ in $(a, b)$
- scalar multiplication is pointwise multiplication: $(cf)(x) = cf(x)$ for each $x$ in $(a, b)$
- if $f$ is bounded and has at most a finite number of discontinuities for $(a, b)$, then $f$ is in $L^2(a, b)$
- if $f$ is unbounded (i.e., has a vertical asymptote in $(a, b)$), then need to check convergence of an improper integral
- the $L$ in $L^2(a, b)$ stands for Lebesgue
  - in calculus, you learn about Riemann integrals
  - Lebesgue integrals are a generalization of Riemann integrals
  - with Riemann integrals, improper integrals are handled as a special case
  - with Lebesgue integrals, improper integrals are handled in the same way as proper integrals
  - Lebesgue integration provides a unified way of handling a larger class of functions than does Riemann integration
  - distinction between Riemann integrals and Lebesgue integrals is not important for this course