Total from length density

1. Charge is distributed on a line segment of length $L$ so that the length charge density is proportional to the square of the distance from one end, reaching a maximum density of $\lambda_0$ at the other end. Compute the total charge on the segment.

   Answer: $Q = \frac{1}{3} \lambda_0 L$

2. Charge is spread on a circle of radius $R$ so that the length charge density varies around the circle. (Note that circle here means the curve as opposed to a disk.) Let $\lambda$ be the charge density measured in Coulombs per meter ($\text{C/m}$). Let $\theta$ measure the angle on the circle from a fixed reference ray (conventionally taken to be the positive $x$-axis). So, the charge density $\lambda$ varies with angle $\theta$.

   (a) Construct a definite integral to compute the total charge on the circle.

   Note: Since we do not yet have a specific density function, we cannot yet evaluate this integral.

   (b) Compute the total charge if $\lambda(\theta) = \lambda_0 (1 + \cos \theta)$ where $\lambda_0$ is a positive constant.

   (c) Compute the total charge if $\lambda(\theta) = \lambda_0 \cos^2 \theta$ where $\lambda_0$ is a positive constant.

   (d) Get a numerical value for the total charge in (c) using the values $R = 0.25 \text{ m}$ and $\lambda_0 = 1.6 \times 10^{-3} \text{ C/m}$.

   Answer: (b) $Q = 2\pi R \lambda_0$  (c) $Q = \pi R \lambda_0$