Integration: a big picture

Fundamental idea of integration: adding up infinitely many infinitesimal contributions to a total

Flavors so far:

- integration over a line segment (i.e., a region in $\mathbb{R}^1$):
  \[ \int_a^b f(x) \, dx \text{ or } \int_{[a,b]} f \, dx \]

- integration over a planar region $R$ (i.e., a region in $\mathbb{R}^2$):
  \[ \iint_R f \, dA \]

- integration over a solid region $D$ (i.e., a region in $\mathbb{R}^3$):
  \[ \iiint_D f \, dV \]
So far, have:

▶ integration over a flat one-dimensional region
▶ integration over a flat two-dimensional region
▶ integration over a flat three-dimensional region

Two ways to generalize:

A. go up in dimension
B. look at non-flat things:

(a) integration over a curve $C$ in plane or space: $\int_C f \, ds$

(b) integration over a surface $S$ in space: $\iint_S f \, dA$

Will focus on B after a few words on A.