Constructing definite integrals

1. Consider the problem of computing the total mass of a column of air. The density of air decreases as height above sea level increases. Let \( h \) be height above sea level measured in meters (m). Let \( \rho \) be the density of air, measured in kilograms per cubic meter (kg/m\(^3\)). Note that \( \rho \) varies with height \( h \). (Here, \( \rho \) is the lower case Greek letter “rho”.)

   (a) Construct a definite integral to compute the total mass of air in a cylindrical column of radius \( R \) and height \( H \) with its base at sea level.

   (b) Compute the total mass of air if \( \rho(h) = \rho_0 e^{-kh} \) where \( \rho_0 \) and \( k \) are positive constants.

   (c) Get a numerical value for the total mass using the values \( \rho_0 = 1.22 \text{ kg/m}^3 \), \( k = 1.1 \times 10^{-4} \text{ m}^{-1} \), \( R = 1 \text{ m} \) and \( H = 10000 \text{ m} \).

2. Consider the problem of computing the total number of bacteria in a circular petri dish. The bacteria colony is more dense at the center than at the edges of the petri dish. Let \( r \) denote radial distance from the center of the dish measured in centimeters (cm). Let \( \sigma \) be the density of the bacteria colony, measured in number per square centimeter (#/cm\(^2\)). Note that \( \sigma \) varies with radius \( r \). (Here, \( \sigma \) is the lower case Greek letter “sigma”.)

   (a) Construct a definite integral to compute the total number of bacteria in a petri dish of radius \( R \).

   (b) Compute the total number of bacteria if the density is \( \sigma_0 \) at the center of the dish and decreases linearly to zero at the edge of the dish.

   (c) Get a numerical value for the total number with the density as in (b) and the values \( \sigma_0 = 5.4 \times 10^3 \text{ per cm}^2 \) and \( R = 5.5 \text{ cm} \).

3. Here is a fact about continuously compounded interest: An amount \( A \) (in dollars) in an account earning interest at a continuously compounded rate \( r \) (in % per year) has a value after \( \tau \) years of \( A e^{r \tau} \).

   Consider the problem of computing the future value of deposits in an investment account. Money is deposited into the account at a known rate and the account earns interest compounded continuously. Let \( t \) be a time in years and \( \delta \) be the deposit rate (in dollars per year). Note that \( \delta \) can vary with time \( t \).

   (a) Construct a definite integral to compute the value of an account \( T \) years in the future.

   (b) Compute the future value if the deposit rate is a constant \( \delta_0 \) in dollars per year.

   (c) Get a numerical value for the future value at 5 years with a constant deposit rate of $1000 per year and an interest rate of 6%. Of this, how much is earned interest?