Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in Some notes on writing in mathematics. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

The project is due in class on Friday, September 23.

For each of the following, determine if the given function is integrable or not integrable for the given interval. Give an argument to support your claim. Base your argument on the definition of integrable in terms of limits and Riemann sums.

Note: The first two problems will account for 85% of the maximum credit.

1. \( f(x) = \begin{cases} 0 & \text{for } x = 0 \\ \frac{1}{x} & \text{for } 0 < x \leq 1 \end{cases} \) for \([0, 1]\)

2. \( f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases} \) for \([0, 1]\)

3. \( f(x) = \begin{cases} \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \text{ in lowest terms} \\ 0 & \text{if } x \text{ is irrational} \end{cases} \) for \([1, 2]\).