The volume element in spherical coordinates

The figure below on the left shows a generic spherical “box” defined as the points with spherical coordinates ranging in intervals of extent \( d\rho \), \( d\phi \), and \( d\theta \). Let \((\rho, \phi, \theta)\) be the spherical coordinates of some particular point in the box. The figure on the right shows a zoomed-in view of the box with the edge lengths labeled \( L_1 \), \( L_2 \), and \( L_3 \).

1. What is \( L_1 \) in terms of \( \rho, \phi, \theta, d\rho, d\phi, \) and \( d\theta \) (as needed)?

2. The length \( L_2 \) is the length of an arc of a circle.
   (a) What is the radius of this circle?
   (b) What is the angle subtended by the arc?
   (c) What is \( L_2 \) in terms of \( \rho, \phi, \theta, d\rho, d\phi, \) and \( d\theta \) (as needed)?

3. The length \( L_3 \) is also the length of an arc of a circle.
   (a) What is the radius of this circle?
      Hint: Draw a vertical cross-section of the sphere.
   (b) What is the angle subtended by the arc?
   (c) What is \( L_3 \) in terms of \( \rho, \phi, \theta, d\rho, d\phi, \) and \( d\theta \) (as needed)?

4. Let \( dV \) be the volume of the box. What is \( dV \) in terms of \( \rho, \phi, \theta, d\rho, d\phi, \) and \( d\theta \) (as needed)?