Problem Statement: Let $f$ and $g$ be complex-valued functions defined as follows:

\[ f(t) = e^{2\pi it} \text{ if } t \in [0, 1], \quad g(t) = e^{2\pi it} \text{ if } t \in [0, 2]. \]

Then,

(a) $f$ and $g$ have the same graph, but are not equivalent, and
(b) the length of $g$ is twice the length of $f$.

Proof. (a) Note that the graphs for both $f$ and $g$ are the same circle on the complex plane, so trivially they are the same graph. Recall that two paths are equivalent if, and only if, there exists some strictly monotonic function $u$ such that $g(t) = f(u(t))$ is a path having the same graph of $f$. So, suppose for contradiction that some $u : [0, 2] \to [0, 1]$ does exist. Then we have

\[ e^{2\pi it} = e^{2\pi iu(t)}, \]

which implies that

\[ \cos(2\pi t) + i\sin(2\pi t) = \cos(2\pi u(t)) + i\sin(2\pi u(t)). \]

So, let $u(1) = c$, and note that $c$ must be in the open interval $(0, 1)$. It follows that $f(c) = g(1)$, and thus $f(c) = 1$. However, this is a contradiction since $1 \notin (0, 1)$, so $f$ and $g$ are not equivalent.

(b) Partition $[0, 1]$ into $n$ partitions of equal size, and partition $[0, 2]$ into $2n$ partitions of equal size. Note that the equal size requirement mandates that one of the partitions on $[0, 2]$ occurs at 1. Since we showed in part (a) that $f$ and $g$ have the same graph it follows that for all points in $[0, 2]$, $f(k) = g(k)$. Thus the inscribed polygon for $g$ must be twice the length of that for $f$. As the number of partitions increases, the ratio of inscribed polygon length holds. Note that this length ratio must additionally hold with arc-length, since inscribed polygon lengths approach the arc-length as the number of partitions increases. Thus, $\Lambda_f(0, 1) = \Lambda_g(0, 1) = \Lambda_g(1, 2)$. By the additive properties of arc-length, it follows that

\[ 2\Lambda_f(0, 1) = \Lambda_g(0, 1) + \Lambda_g(1, 2) = \Lambda_g(0, 2), \]

proving our claim. \qed