Fourth Modeling Problem

Due Dates:
- Draft Report: Monday, March 28th
- Peer Reviews: Wednesday, March 30th
- Final Report: Friday, April 1st

In class we have been discussing probability models. We have considered both discrete models and continuous models, and while we have not gone into significant depth, we have covered such basic ideas as distributions, independence, expectation, and variance for both kinds of models. Elements of these topics will probably come in handy as you work on this week's modeling project.

For this modeling assignment, you can choose from five possible topics. All these topics are based on UMAP Modules. (UMAP stands for “Undergraduate Mathematics and Its Applications”, and a “UMAP Module” is a peer-reviewed, guided project, generally with exercises, that has been published in the UMAP journal.) Your task is to choose a module that interests you, use it to identify a problem, and tackle that problem with whatever means you can. These means can include any of the theory worked out in class or in the module, but can also extend beyond the module, if you wish to pursue some particular thread further and in greater depth.

Part of the challenge with this assignment is to form an appropriate problem statement. Another part is to use appropriate modeling techniques. The exercises embedded within the module can help you with both tasks: an exercise is often a “research question” in disguise, and solving the exercises helps you figure out how to use certain techniques. Since the answers to the exercises are generally given, your task in some sense reduces to formulating a good question and providing a good exposition of how to answer that question. But there is also plenty of leeway for creative investigation, if you have the appetite and/or time to pursue it. (The best papers will pursue such independent avenues of thought.)

Please note that your draft report is due one week from today. This entire week of class will be devoted to working with your group. I propose that you quickly decide on which line of thought you’d like to pursue, and come to class tomorrow having read the relevant UMAP module and prepared to make progress.

The modules from which you can choose are the following:

1. **UMAP Module 311.** This module is called “Geometry of the Arms Race”, and explores a game-theoretic way to think about ending the arms race.

2. **UMAP Module 327.** This module is called “Nuclear Deterrence”. The introduction says:

   This module is designed to apply mathematical models to nuclear deterrent problems, and to aid users in developing enlightened skepticism about the use of linear models in stability analyses and long-term predictions. An attempt is made at avoiding overwhelming complexities through concentration on land-based missile forces. It is noted that after the Second World War, the United States’ monopoly on nuclear weapons permitted it to attempt to deter aggression through a policy of “massive retaliation.” The history of change from this policy is outlined, and the chances for reaching an equilibrium in the nuclear arms race are examined through probabilistic models.

3. **UMAP Module 340.** This module is called “The Poisson Random Process”. Probability distributions are introduced to obtain practical information on random arrival patterns, interarrival times or gaps between arrivals, waiting line buildup, and service loss rates. The Poisson distribution, the exponential distribution, and Erlang’s formulas are used. Two problems are posed at the outset. They are:
**Problem A:**
Suppose that you live in an isolated community where fires break out at random at an average of 3 per day. If fires require an average of 1 hour to fight, how many firefighting units should your fire station have to make the community “safe?”

**Problem B:** Suppose that you own a hardware store that carries brooms. Your merchandise is restocked only at the close of your business week, each Saturday afternoon. You have limited storage space and therefore wish to keep inventory levels at a minimum. If customers who buy brooms arrive at random times and at an average rate of 10 per week, how many brooms should you have on hand each Monday morning?

4. **UMAP Module 738.** This module is called “The Hardy-Weinberg Equilibrium.” It deals with a central topic in population genetics. The abstract states:

   Using elementary probability, this unit shows how genotypes in a population reach an equilibrium in a single generation, under appropriate conditions. Real data illustrate this concept, and conditions are examined that may keep a population from attaining an equilibrium.

5. **UMAP Module 792.** This module is called “The Spread of Forest Fires.” It uses probability to model conservation tactics. The abstract states:

   We create a simple discrete probabilistic model for spread of a forest fire. We examine the conditions for which the fire will either die out or spread indefinitely, identifying and bounding a critical value for the probability of transmission of the fire to an immediately adjacent location.