1. Suppose an animal lives three years. The first year it is immature and does not reproduce. The second year it is an adolescent and reproduces at a rate of 0.8 female offspring per female individual. The last year it is an adult and produces 3.5 female offspring per female individual. Further suppose that 80% of the first year females survive to become second-year females, and 90% of second-year females survive to become third-year females. All third-year females die. We are interested in modeling only the female portion of this population.

(a) Draw a state diagram for this scenario.
(b) Construct the Leslie matrix.
(c) Compute the eigenvalues for this matrix. From these determine if the population will eventually grow or decline. What is the rate of this growth (or decline)?
(d) Suppose that a population of 100 first-year females are released into a study area along with a sufficient number of males for reproductive needs. Track the female population over 10 years.

2. Showing that a model is valid can be difficult. Often it is much easier to show that a model is invalid. Consider the following model that has been proposed to describe the growth of redwoods. Redwoods frequently live 1000 years or more. The model used classifies redwoods into three age stages: 0 to 200 years (young), 200-800 years (mature), and older than 800 (venerable). A current census in a particular stand finds that currently there are 1,696 young redwoods, 485 mature redwoods, and 82 old redwoods. The transition between stages over a 50-year period is given by the matrix

\[
T = \begin{pmatrix}
0.12 & 0.26 & 0.6 \\
0.30 & 0.92 & 0 \\
0.18 & 0.18 & 0.67
\end{pmatrix}
\]

Trace this stand of trees through five time steps (five 50-year periods for a total of 250 years) and explain how you know the model cannot possibly be valid.

3. A copy machine is always in one of two states, either working or broken. If it is working, there is 70% chance that it will be working tomorrow. If it is broken, there is a 50% chance it will still be broken tomorrow. Assume that one day is a natural time step.

(a) Draw a state diagram for this scenario.
(b) Formulate the transition matrix.
(c) Assuming that the machine is working today, what is the probability that it will be working tomorrow? The next day? After one week? After one month?
(d) What is the long-term probability that the copy machine will be working on any given day?

4. A slightly more refined model of a copy machine has three states: working, broken and fixable, broken and unfixable. If it is working, there is 69.9% chance it will be working tomorrow and a 0.1% chance it will be broken and unfixable tomorrow. If it is broken and fixable today, there is a 49.8% chance it will be working tomorrow and a 0.2% chance it will be unfixable tomorrow. Unfixable is, of course, unfixable, so the probability that an unfixable machine is unfixable tomorrow is 1 and the probability of its being anything other than unfixable is 0. Again, assume a day as the time step.

- Draw a state diagram for this scenario. Categorize the states as absorbing and non-absorbing.
- Formulate the transition matrix. Label the states so that the identity matrix is in the lower right-hand block.
- Compute the fundamental matrix and interpret the results. How long will this machine last? How much of that time will it be working, and how much of that time will it be under repair (broken and fixable)?