Introduction:

Linear algebra is arguably the single most important subject within the entirety of the undergraduate mathematics curriculum. Part of the reason for this importance is the relevance of the subject outside of mathematics: linear equations play important roles in a wide range of quantitative modeling applications, and subjects as diverse as physics, biology, chemistry, economics, sociology, and computer science all make hefty use of linear algebra. But another reason for the importance of the subject has to do with its paradigmatic role within mathematics. Many of the ideas, objects, and techniques that have been developed to understand linear systems form the core ideas and key examples behind a wide range of higher-level mathematics. A strong understanding of linear algebra will thus serve you well, whether you continue on in your study of mathematics or advance in some other quantitative discipline.

At the University of Puget Sound, linear algebra also plays the role of a “gateway” course: all mathematics majors need to take it, and in addition to learning the nuts and bolts of the subject, students are also supposed to learn the black art of writing mathematical proofs. The proof writing element is specifically designed to give mathematics majors the basic argumentative and expository skills they will need to succeed in higher-level mathematics courses. It is worth noting that the stylistic and logical elements that go into writing good proofs are similar to those that go into writing in general, and thus the proof-writing training should serve you well regardless of your major.

In this course you will learn to work like a mathematician. While “success” in some mathematics classes might reduce to answering procedural questions correctly, here “success” means that you will learn to pose your own questions, debate with your peers, teach yourself new material, and critically analyze mathematical arguments. As a means to the lofty end of “working like a mathematician”, this course will be taught within an “inquiry based learning” framework. Details about this framework are included below, but in nutshell, it means this: that you are responsible for teaching yourself new material outside of class, and that class time will for the most part be reserved for discussion, argumentation, and enrichment. In particular, there will be very little lecture (if any) in this course, and you will be expected to take an active role not just in writing up solutions, but also in communicating them with your peers.

Finally, in addition to learning the nuts and bolts of the subject, the art of proof writing, and the art of mathematical inquiry, you will learn some things about software in this course. Mathematicians
generally typeset their arguments using a programming language called Latex. You’ll become proficient in this software over the course of the term. You’ll also learn how to use Sage, a Computer Algebra System (CAS) that can streamline many of the ugly computations that sometimes characterize linear algebra. I think you will find that both your Latex and Sage skills will be useful in academic and professional settings beyond this class.

Linear algebra is a powerful, rich, and exciting subject. This course will involve a lot of work, both on your end and on mine, but it will be an exciting journey, and one well worth the effort!

**Course Catalog Description:**

This course is a study of the basic concepts of linear algebra, and includes an emphasis of developing techniques for proving theorems. Students will explore systems of linear equations, matrices, vector spaces, bases, dimension, linear transformations, determinants, eigenvalues, change of basis, and matrix representations of linear transformations.

**Specific Learning Goals:**

After successfully completing this course, you should be able to:

1. Solve a range of low- and high-level linear algebra problems:
   a. Low-level: mostly computational problems involving matrices and vectors.
   b. High-level: computational or theoretical problems involving such concepts as the three pictures, the four central problems, explicit vs. implicit descriptions, the Fundamental Theorem of Linear Algebra, etc.
2. Communicate mathematics effectively in both written and oral form.
3. Use Sage and Latex effectively and comfortably.
4. Produce well-written, finely argued mathematical proofs.

More broadly, this class should improve your capacity to think critically about where, how, and why linear algebra is used, to become a deft problem solver, and to become an effective technical communicator.

**Required Text:**

- A First Course In Linear Algebra, by Rob Beezer. (Open source, and available at http://linear.ups.edu/html/fcla.html.)

The course will cover roughly the first seven chapters of Strang’s text, and the corresponding material in Beezer’s.

**Other Requirements:**

- A laptop that you can bring to class for interactive computing sessions. (If you don’t have one, let me know and we will try to find a work-around.)

**Assessment:**
I plan to use a *standards based assessment strategy* to assess your performance this term. In essence, a standards based assessment scheme defines a set of specific topics that need to be mastered. In this class, I have 14 standards, 10 of which are “fundamental”, and 4 of which are “advanced” (see below for details.) Your goal is to show me that you have mastered these topics, and you will do this by taken a written assessment. I’ll give these assessments about once every two weeks, two standards per assessment session. Once you have mastered a “standard”, it is logged in my grade book as “passed” and you don’t need to deal with it again. If you don’t pass a standard, you can sometimes try it again later on. (For example, on the final exam.)

In addition to your performance on the “standards” assessment, your grade will be based on the following elements:

- **Daily homework**: every day, you will have a reading assignment, and a number of “Tasks” to complete. These need to be written out carefully and brought to class to be used as the basis of discussion. You will then turn them in and I will grade them on a pass/fail basis. Work that is detailed, clear, and shows an honest attempt at every Task will generally pass; work that is sloppy, thoughtless, or incomplete will generally not pass.

- **Bi-weekly proofs**: every two weeks, I will assign a new set of proof problems. Proofs need to be written up in Latex and submitted in hard-copy. Proofs too will be graded on a pass/fail basis. Proofs that are correct, well-written, free of typos and that employ mathematical notation correctly will generally pass; proofs that violate one or more of these features will generally not pass. A proof that fails can be re-attempted at the next submission window.

- **Presentations**: each day in class we will discuss the Tasks of the previous night. Most days will begin by having you get into groups to discuss your solutions, and then having the groups send a representative to the board to present a “common” solution to the class at large. You are expected to volunteer for these presentations, on a regular basis. We’ll spend some time discussing what a presentation should look like, but in general, they should be fluid, articulate, and correct. You will get credit for every successful presentation.

**Grading scheme:**

The grading scheme I propose will look something like the following:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Foundational Standard</th>
<th>Advanced Standard</th>
<th>Daily Homework</th>
<th>Proofs</th>
<th>Presentations</th>
<th>Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>4</td>
<td>&gt;90%</td>
<td>13</td>
<td>6</td>
<td>pass</td>
</tr>
<tr>
<td>A-</td>
<td>10</td>
<td>3</td>
<td>&gt;90%</td>
<td>12</td>
<td>6</td>
<td>pass</td>
</tr>
<tr>
<td>B+</td>
<td>9</td>
<td>2</td>
<td>&gt;80%</td>
<td>11</td>
<td>4</td>
<td>pass</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>1</td>
<td>&gt;80%</td>
<td>10</td>
<td>4</td>
<td>pass</td>
</tr>
<tr>
<td>B-</td>
<td>8</td>
<td></td>
<td>&gt;80%</td>
<td>9</td>
<td>4</td>
<td>pass</td>
</tr>
<tr>
<td>C+</td>
<td>7</td>
<td></td>
<td>&gt;70%</td>
<td>8</td>
<td>2</td>
<td>pass</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td></td>
<td>&gt;70%</td>
<td>7</td>
<td>2</td>
<td>pass</td>
</tr>
<tr>
<td>C-</td>
<td>5</td>
<td></td>
<td>&gt;70%</td>
<td>6</td>
<td>2</td>
<td>pass</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td></td>
<td>&gt;60%</td>
<td>5</td>
<td>1</td>
<td>pass</td>
</tr>
<tr>
<td>F</td>
<td>0-2</td>
<td></td>
<td>0-59%</td>
<td>0-4</td>
<td>0</td>
<td>fail</td>
</tr>
</tbody>
</table>
I reserve the right to modify this table to accommodate our actual progress (which may differ from our predicted progress.)

**Definition of Standards:**

I have stolen these standards verbatim from Theron Hitchman, a professor of mathematics at Iowa State University. (Dr. Hitchman is also the author of the IBL notes that we are using.) Each standard is in essence a response to a particular question. The level of the question, along with the complexity of the tools that you are expected to use to answer it, determines whether the standard is “Fundamental” or “Advanced.” Note that a given question can be answered using a variety of tools, and the same tool can be used to answer a variety of questions.

**Foundational Standards**

**Question One: What are the basic objects of linear algebra?**
- Vector Algebra (Chapter 1)
  - add vectors, plot vectors, compute scalar multiplication of number and vector, compute linear combinations, geometric interpretations of these operations
- Matrix Algebra (Chapter 1 and 2)
  - add matrices, take transpose, multiply matrix times vector (two ways), multiply two matrices (three ways), identify troubles with matrix multiplication: commutativity, inverses
- The Dot Product (Chapter 1)
  - compute the dot product of two vectors, compute angles between vectors, compute length of a vector, normalize a vector, use connection between dot product and linear equations to work with normal vectors

**Question Two: How can we solve a square system of linear equations?**
- Gauss-Jordan Elimination (Chapter 2)
  - Use Gauss-Jordan and back-solving to solve a system, find LU decomposition, identify when Gauss-Jordan breaks, identify when matrix does not have an LU decomposition and discuss workaround, compute determinant of a square matrix, compute the inverse of a square matrix

**Question: How can we solve a general system of linear equations?**
How can we tell if there is a solution? What shape will the solution set have? When will the solution be unique? Is there a computationally effective way to find the solution set?
- Solving Systems of Equations (Chapter 3)
  - Solve a general (rectangular) system of linear equations using the reduced row-echelon form, special solutions, a particular solution. Give the general solution to a system of linear equations. Compute the rank of a matrix. Use pivots and free variables to reason about the solution set to a system of equations
- The Four Subspaces (Chapter 3)
  - Compute the null space, column space, row space, and left null space of a matrix. describe these subspaces by giving bases

**Question: What are the good ways to understand subspaces?**
- Implicit and Explicit Descriptions (Chapter 3)
  - determine when a set of vectors is linearly dependent or linearly independent, determine the span of a set of vectors. determine if a collection of vectors is a basis for a subspace Find a basis for a subspace described using equations, find equations
to describe a subspace described using a basis use the row space algorithm and the column space algorithm to find a basis

**Question:** Can we find approximate solutions to systems that do not have an actual solution?
- **Approximate Solutions and Least Squares (Chapter 4)**
  Find the “best” available approximate solution to an unsolvable system of equations, draw pictures explaining how orthogonal projection is relevant, use approximate solutions to fit curves to data

**Question:** Is there a good way to test if a square matrix is invertible?
- **Determinants and the Invertible Matrix Theorem (Chapter 5)**
  Use properties of determinants, compute determinants in a variety of ways, Use the invertible matrix theorem

**Question:** How can we understand the geometry of square matrices as transformations?
- **Eigenvalues, Eigenvectors, and the Spectral Theorem (Chapter 6)**
  Compute eigenvalues and eigenvectors, diagonalize matrices when possible (and recognize limitations), know and use spectral theorem

**Advanced Standards**

**Question:** Are there any good geometric interpretations of a system of linear equations?
- **The Three Viewpoints (Chapters 1 and 2)**
  The row picture, the column picture, and the transformational picture. pass back and forth cleanly pass between the representations, and describe what a solution means in each case.

**Question:** How do we understand matrices as transformations?
- **Four Subspaces and Matrices as Functions (Chapters 3 and 4)**
  Use the four subspaces to describe the action of a matrix as a transformation (function) Draw reasonably accurate schematic of the transformational picture using information about the four subspaces, make conclusions about the nature of a matrix using the four subspaces

**Question:** Is there a way to choose a geometrically good basis for a subspace?
- **Orthonormal Bases and the QR Decomposition (Chapter 4)**
  Use Gram-Schmidt to compute an orthonormal basis for a subspace, decide if a matrix is orthogonal or not, compute the QR decomposition of a matrix

**Question:** Is there a good geometric way to understand the behavior of a general matrix as a function?
- **Singular Value Decomposition (Chapter 6)**
  Compute the SVD of a matrix, reason about the structure of a matrix using the SVD, build matrices with given properties using the SVD.

**Class Expectations:**

I try to structure a classroom that revolves around the twin pillars of *discovery* and *discussion*. I believe that learning is an active process, and that class works best when it functions as an opportunity for guided inquiry, where the “guides” are both the professor and the peer group. In the wider educational community, my approach is labeled “Inquiry Based Learning”, or IBL.

There are a number of ways to structure an IBL class, but salient features of this class will include the following:
- Lectures replaced by hands-on activities
• An emphasis on group work
• Lots of dialogue and discussion
• Student presentations
• A supportive environment in which to take “risks”
• An emphasis on communication, both oral and written
• A need for trust and confidence, both student-student and student-professor

Productive failure is an idea that lies at the root of our approach. When you’re trying to learn something, never making a mistake is generally a sign that you’re not pushing yourself hard enough. This class should be a safe and supportive space in which to get things wrong. When talking or presenting, you are challenged to work slightly outside of your comfort zone, to volunteer answers when you have a pretty good idea but aren’t 100% certain, to risk a conjecture that might turn out to be off the mark. And when you are listening to fellow students talk, you are challenged to pay strict attention, to flag small errors of language or comprehension, and to politely and respectfully help guide one another towards a clearer and truer picture of the matter at hand. Failure is part of the design spec for this class, and it can be hard, but you will not be struggling alone.

Although the spirit of what I’m shooting for with this IBL style class is probably clear, here is a minimalist list of concrete expectations:

• attend class daily
• do all assigned homework
• do all assigned reading, in a timely fashion
• participate actively in class discussions and class group work activities
• volunteer to present solutions on the board
• volunteer answers to questions I pose, and to ask your own questions when you are confused, uncertain, or simply thinking outside the box
• be courteous and supportive of your fellow learners
• help create a classroom that is a supportive, energetic, respectful place to learn.

More broadly, my basic hope and expectation is that you will engage the spirit of Inquiry Based Learning with enthusiasm, openness, and joy (it is fun), and help make this class a fun and supportive place in which to learn.

Lastly, a word about goals and outcomes: the goal in IBL is to produce critical thinkers who have a strong, creative command of the subject material. There is ample research evidence to support the IBL model, and I’m happy to share it with you if you’d care to see it. For me, one of the strongest element of IBL is the host of secondary skills you develop almost “for free”, including skills in abstract thinking, working from first principles, working with other people, and communicating your ideas. As my colleague Dana Ernst has noted, “All of the secondary skills you will develop in this course are highly valued by society. Whether you become a teacher, a lawyer, an engineer, or an artist, what differentiates you from your competition is your ability to think critically at a high level, collaborate professionally, and communicate effectively.”

Policies:

Attendance:

Built into the philosophy of inquiry-based learning is the idea that we help one another to learn. As a consequence, your daily attendance in class is very important, not just for your own benefit, but for that of your peers.
I will take role every day and verify attendance. You are allowed to miss up to three days with no penalty. After three missed classes, two things happen: 1) every additional absence will reduce your grade by one level, 2) I reserve the right to drop you from the class.

**Late work:**
In this class, I will not accept late work. (In the case of extreme emergency, talk to me, and can probably make an exception.) Note on the grading scale that you are allowed to miss a couple of proofs and daily homeworks and still get a good grade, so if you are late with an assignment, simply consider it one of the “missed” ones, and work hard to get the next one in on time.

**Planned absences:**
If you need to be absent for some planned family or medical reason, you should contact me in advance. If an emergency arises, contact me as soon as possible after the emergency has passed. Student athletes who need to miss class for games should let me know of this as early as possible.

**Classroom policies:**
You are welcome to bring a cup of coffee or a bottle of water to class, but please eat your meals outside of class. Please turn off your phones and keep your laptops closed, unless we happen to be doing a computer exercise. You can take a bathroom break if necessary, but please make this the exception, not the rule—in general, I don’t want people entering and leaving the room during class.

**Academic integrity:**
It is your responsibility to understand the academic integrity policy of the university. You can find this policy in the Academic Handbook, and it is also available online at:
Not citing other people's work, turning in the same work to satisfy two different classes, citing false information, or plagiarism are all violations of the academic integrity policy. Such violations are taken very seriously, and will be reported if discovered.

**Disabilities:**
If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Peggy Perno, Director of Student Accessibility and Accommodation, 105 Howarth Hall, 253-879-3395. She will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.

**Classroom Emergency Response Guidance**
Please review university emergency preparedness and response procedures posted at www.pugetsound.edu/emergency/. There is a link on the university home page. Familiarize yourself with hall exit doors and the designated gathering area for your class and laboratory buildings.

If building evacuation becomes necessary (e.g. earthquake), meet your instructor at the designated gathering area so she/he can account for your presence. Then wait for further instructions. Do not return to the building or classroom until advised by a university emergency response representative.
If confronted by an act of violence, be prepared to make quick decisions to protect your safety. Flee the area by running away from the source of danger if you can safely do so. If this is not possible, shelter in place by securing classroom or lab doors and windows, closing blinds, and turning off room lights. Lie on the floor out of sight and away from windows and doors. Place cell phones or pagers on vibrate so that you can receive messages quietly. Wait for further instructions.

**Getting Help:**

Linear algebra is an advanced class, and there is not official tutoring for this subject. Your first line of attack should be your peers: we’ll work on building peer groups in class, and get in the habit of trying to work with classmates outside of class. I’m around: I have regular office hours, and if you can’t make those, then try just dropping by or setting up an appointment (I am generally in my office if I’m not teaching, eating, or working out.) Make a habit of coming to see me to discuss any questions you might have. I imagine that one of the reasons you chose to go to a liberal arts college was to gain access to your professors, and I am very much on board with that vision. You should leverage me as a resource.