Example Proofs

**Task:** Examine the following statement and two "proofs" that follow. Which proof is better? Why? Discuss. Then provide your own proof of the theorem.

**Statement.** For any positive integer $n$, if $n^2$ is a multiple of 3, then so is $n$.

**Proof A.** Assume that $n^2$ is an odd positive integer that is divisible by 3. That is $n^2 = (3n + 1)^2 = 9n^2 + 6n + 1 = 3n(n + 2) + 1$. Therefore, $n^2$ is divisible by 3. Assume that $n^2$ is even and a multiple of 3. That is,

$$n^2 = (3n)^2 = 9n^2 = 3n(3n).$$

Therefore $n^2$ is a multiple of 3. If we factor $n^2 = 9n^2$, we get $3n(3n)$; which means that $n$ is a multiple of 3.

**Proof B.** Suppose to the contrary that $n$ is not a multiple of 3. We will let $3k$ be a positive integer that is a multiple of 3, so that $3k + 1$ and $3k + 2$ are integers not multiples of 3. Now $n^2 = (3k + 1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$. Since $3(3k^2 + 2k)$ is a multiple of 3, $3(3k^2 + 2k) + 1$ is not. Now we will do the other possibility, $3k + 2$. So, $n^2 = (3k + 2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$ is not a multiple of 3. Because $n^2$ is not a multiple of 3, we have a contradiction.

*Your proof:*