Quiz 5

1. Suppose you wish to estimate the mean glucose level of a certain population of fasting rats. You happen to know that the standard deviation of glucose levels is 6. You measure the glucose levels of 9 rats, and find that the average of these measurements is 243. Give a 95% confidence interval for the true mean glucose level.

\[ \sigma = 6 \quad \bar{x} = 243 \]

95% C.I.: \[ \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \], where \[ z^* = 1.96 \]

So: \[ 243 \pm 1.96 \left( \frac{6}{\sqrt{9}} \right) = 243 \pm 1.96 \left( \frac{2}{1} \right) = \frac{243 \pm 3.92}{2} \]

\[ = [239.08, 246.92] \]

Alternative: Instead of using \[ z^* = 1.96 \] (which comes from \[ N(0,1) \] table), use \[ \sigma = 2 \] (which comes from \[ 68-95-99.7 \% \] Rule). In this case, the C.I. is \[ 243 \pm 2 \cdot \frac{6}{\sqrt{9}} = (243 \pm 4) = [239, 247] \]

2. Suppose you take an SRS of size 25 from a population of skunks, and measure their stripe lengths. Your sample average is 10cm. Suppose you happen to know that the standard deviation of stripe lengths is 2cm. Calculate the \( P \)-value of your data if the null hypothesis is \( H_0: \mu = 11 \) and the alternative hypothesis is \( H_a: \mu \neq 11 \). Can you reject the null hypothesis at the .05 level of significance?

\[ \bar{x} = 10 \]

\[ \sigma = 2 \]

\[ H_0: \mu = 11 \quad n = 25 \]

\[ H_a: \mu \neq 11 \]

First form the test statistic:

\[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{10 - 11}{2/\sqrt{25}} = -1 \cdot \frac{5}{2} = -2.5 \]

Then calculate the area in two tails:

Using a \( N(0,1) \) table, the answer is \[ 2 (0.0124) = 0.0248 \]

Since \[ .0248 < .05 \], we do reject \( H_0 \) at the 5\% level.
3. True or false: if someone tells you that a 95% confidence interval for the percent of Americans that support Obama is 45 ± 2, then you can conclude that the true percent of Obama supporters lies in the interval [43, 47] 95% of the time. (Justify your response.)

False. The true percent of Obama supporters is fixed and does not vary from sample to sample. So it either is in [43, 47] or it is not.

4. True or false: if someone tells you that they did a hypothesis test and got a P-value of .005, then you can conclude that probability that the null hypothesis is true is .005. (Justify your response.)

False: a P-value of .005 means that the probability of getting data “as or more extreme” than what you got is .005, assuming the null hypothesis were true.