Guide to Writing Proofs
Math 375, Toews, Spring 2014

Mathematics majors at Puget Sound are expected to gain proficiency in mathematical writing by taking a number of upper division courses that emphasize proofs. Math 375 is one such course. As participants in this course, you will be expected to produce a considerable body of well-crafted, logically correct and aesthetically compelling mathematical proofs over the course of the semester. This guide sketches out what I expect to see in your writing, and how your work will be evaluated.

1. About mathematical writing

The purpose of writing a mathematical proof is to convince the reader that the accompanying theorem is correct. Logic is the principle tool in this task, but not the only one: readers are human beings, and the proof needs to be tailored to the human audience. Accordingly, proofs should be not just logically correct but also notationally clear, grammatically polished, and structurally elegant.

Since most of these features characterize good writing in general, it is tempting to view a proof as a kind of “mini essay”. In some sense, this is a valuable viewpoint: it emphasizes the important idea that the proof needs to convince, not just step through a sequence of incontrovertible claims. But there some issues that arrive in mathematical proof writing that are unique to the discipline, and need to be thought about as you write:

- **Notation**: Notation should be clear and unambiguous. Use orthographically distinct symbols for ontologically distinct entities. Make sure there is a logic to your choice of variables, and that the logic is clear to the reader. Define every variable you use.

- **Equations**: Equations are part of sentences, and should be punctuated accordingly. You can do this “inline”, by writing, e.g., *the equation* $E = mc$ *has played an important role in history*, or in “display” mode, e.g. *the equation*

  $E = mc^2$

  *has played an important role in history*. If an equation ends a sentence, give it a period. (Caution: it is easy to forget to do this in display mode. Pay attention.) Equations that are displayed can be numbered, but should only be numbered if they are referred to later on. Having very long lists of displayed equations is generally considered bad form (if your list has more than about three lines, consider breaking it up.)

- **Handwaving**: Frequently, you will want to invoke some fact that seems obvious to you, but which you have not proved. Vague keywords like sometimes allow you to do this, e.g. “*Since every matrix of rank 0 clearly is injective, we see that....*” You need to make a judgement call about how “clear” such things are to the audience–if the audience needs to do too much work to follow or check your proof, you have not proved the result.

- **Logic**: In general, your argument should follow the shortest possible path to the conclusion. Including facts or arguments that don’t directly support your case is misleading and poor form.

- **Grammar**: Write in complete sentences that are correctly punctuated. Never begin a sentence with a mathematical symbol.

- **References**: Make sure you cite every non-trivial result you use without proof. (In general, you won’t be doing this in the class, but if you do, include a citation.)

2. How I will grade your work

Every proof-problem that you submit to me will be graded on a 10-point scale. Five of these points pertain exclusively to its logical and mathematical properties, two pertain to notation and terminology, and three are reserved for stylistic issues. My rubrics (which I filched from Jason Preszler, with permission) will be as follows:
3. Writing in Latex

I expect all of your work to be typeset in Latex, a program specifically geared towards writing mathematics. There is a learning curve to Latex, but it’s not too bad, and I will help you if/when you need help.

There are several options for how to access and use the Latex software. One good option is to download the software to your personal computer. This is a good choice because then you can write mathematics from wherever you happen to be, even if you don’t have web connectivity. The software can be downloaded from the following sites:

- [http://tug.org/mactex/](http://tug.org/mactex/) (for Mac)

There are also websites that you can log into and write Latex from. These options are nice because you don’t need to worry about installing or updating anything, and they keep a copy of your files (i.e. you get free backup.) They are also great if you ever need to share documents. Two sites worth checking out:

- [https://www.sharelatex.com/](https://www.sharelatex.com/)
- [https://www.writelatex.com/](https://www.writelatex.com/)

Most of you already know basic Latex syntax. If you need to learn it, or just want a refresher, I recommend working through a tutorial. If you google “latex tutorial” you will find lots of these things. Here are some that look promising:

- [http://www.tug.org/twg/mactex/tutorials/ltxprimer-1.0.pdf](http://www.tug.org/twg/mactex/tutorials/ltxprimer-1.0.pdf)

Finally, even if you understand the basics of Latex, there are always things you will need to look up (e.g. how do you write an equation that extends beyond one line? How do you align three equations and only put a reference number on the second one? Etc.) You’ll probably want some references. Two good ones are the following:

- [http://detexify.kirelabs.org/classify.html](http://detexify.kirelabs.org/classify.html) (allows you to draw a symbol, and find the name)