Project 2

The objective of this project is to get you to started thinking about how models emerge from data, and to practice using your various methods of analysis to say something significant about the underlying physical system.

The first chapter in our text has been both an introduction to model building, and a rough primer on “things you can do once you have a model.” Recall that model building can essentially be considered a three part process:

1. State assumptions on which to base the model
2. Describe the variables and the parameters of the model
3. Based on the assumptions, derive appropriate differential equations using your variables and parameters.

Once you have a model, there are various ways of analyzing it. Sometimes (if you’re lucky) you can actually “solve” the differential equation and get an analytic expression for a function $y(t)$. Even if you can’t solve the differential equation, you can analyze the model qualitatively using a variety of techniques, including forming slope fields and phase lines, discussing when solutions exist and when they are unique, identifying the domain of solutions and if/where they blow up, analyzing the limiting behavior of solutions as time gets large, etc. For some of these tasks a computer is a critically important tool.

The starting point for this assignment is the following data set giving estimated total biomass of the north Atlantic cod population every year for the last ten years:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catch (Tons)</td>
<td>1.64</td>
<td>3.03</td>
<td>5.63</td>
<td>4.40</td>
<td>6.07</td>
<td>6.68</td>
<td>6.05</td>
<td>8.25</td>
<td>9.28</td>
<td>7.42</td>
</tr>
</tbody>
</table>

The numbers reflect the total population under a “fixed catch” policy that caps total commercial cod intake at 0.5 metric tons per year. Prior to the advent of this policy, population levels had been decimated by overfishing. The data clearly suggests that levels have rebounded, but there has been a dip in the last year, and fishery managers are wondering what the implications of this dip might be. Questions they are asking include:

- What is the maximum carrying capacity of the ecosystem with no fishing?
- Has the steady state equilibrium under the current catching regime already been reached? If so, what exactly is it?
- Could the population have peaked and be on the cusp of another long descent?
- Could the catch level be increased without destroying the fish population? If so, by how much?
- Is there a critical depensation level, i.e. a level below which the population cannot sustain itself?

**The assignment:** Your task is to model this cod fishery using a model of the form

$$\frac{dy}{dt} = ay\left(1 - \frac{y}{b}\right)(y - 1) - c,$$
where

\[ y(t) \]  total biomass of cod at time \( t \).

\[ a \]  growth rate

\[ b \]  carrying capacity

\[ c \]  catch rate

Your task is to prepare a brief report for fisheries managers that addresses their concerns and summarizes your findings.

Some food for thought, and some directives:

• Can you solve the differential equation analytically?

• What can you say qualitatively about solutions?

• Is a big decline possible, given the model?

• Do you think random noise (i.e. fluctuations) in the population from year to year might lead to collapse under the present values of \( y \) and \( c \)?

• How can you find the best values for \( a \) and \( b \), given that you know \( c \) (at least over the span of the data set?)

• Which is a more important variable from a policy perspective, \( a \) or \( b \)?

• Are solutions to this DE unique? Over what time span can they be expected to exist?

**Plots:** Your work can (and should) contain one or more plots. In thinking about plots, think about what kind of information would be useful to somewhat trying to regulate fishing policy (i.e. determine the best value of \( c \)) Include at least one plot that shows \( y(t) \) for the next 10 years under three choices of \( c \) (one high, one medium, one low.) Also show at least one plot that superimposes the data over the curve predicted by your best estimate of \( a \) and \( b \).

This project is due **Tuesday, March 12th**, at the beginning of class.