Quiz 8

(1) Calculate \( \int_{0}^{1} \int_{-1}^{1} y \sin x \, dx \, dy \)

\[
\int_{0}^{1} \int_{-1}^{1} y [ \int_{-1}^{1} \sin x \, dx ] \, dy
\]

\[
= \int_{0}^{1} y [ -\cos x ]_{-1}^{1} \, dy
\]

\[
= \int_{0}^{1} y \cdot 0 \, dy
\]

\[
= \int_{0}^{1} 0 \, dy
\]

\[
= 0
\]

(2) Sketch the region enclosed by the lines \( y = 2, y = 2 - x, \) and \( y = x - 2. \) Then calculate the integral of \( f(x, y) = xy^2 \) over this region.

\[
V = \int_{0}^{2} \left[ \int_{x-2}^{2-y} x y^2 \, dx \right] dy
\]

\[
= \int_{0}^{2} \frac{(2+y)^3}{3} - \frac{(2-y)^3}{3} \, dy
\]

\[
= \frac{1}{2} \int_{0}^{2} 8y^3 \, dy
\]

\[
= \frac{1}{2} \left[ \frac{y^4}{4} \right]_{0}^{2}
\]

\[
= \frac{1}{16}
\]

(3) Today's quiz is very short. Produce a short, artistic expression of dismay at the brevity of life.