Quiz 3

In the following, let \( P = (1, 0, 0), Q = (1, 1, 0), \) and \( R = (0, 1, \sqrt{3}) \).

(1) Find an equation of the plane that passes through \( P, Q, \) and \( R \).

\[
\overrightarrow{PR} = R - P = (-1, 1, \sqrt{3})
\]
\[
\overrightarrow{PQ} = Q - P = (0, 1, 0)
\]
\[
\overrightarrow{PR} \times \overrightarrow{PQ} = \begin{vmatrix}
 1 & i & k \\
 -1 & j & \sqrt{3} \\
 0 & 0 & 0
\end{vmatrix} = i(-\sqrt{3}) - j(0) + k(-1) = (-\sqrt{3}, 0, -1)
\]

given plane \( \frac{-\sqrt{3}x - z = d}{\sqrt{3}} \)

Since \( P \) lies on plane:

\[-\sqrt{3}(1) - 0 \cdot 0 = d \implies d = -\sqrt{3}\]

Final equation: \( -\sqrt{3}x - z = -\sqrt{3} \)

\[
\begin{align*}
\| \overrightarrow{PR} \| &= \sqrt{(-1)^2 + 1^2 + (\sqrt{3})^2} = \sqrt{5} \\
\| \overrightarrow{PQ} \| &= \sqrt{0^2 + (1)^2 + (0)^2} = 1 \\
\| \overrightarrow{PR} \times \overrightarrow{PQ} \| &= \sqrt{(-\sqrt{3})^2 + 0^2 + (-1)^2} = \sqrt{3 + 1} = \sqrt{4} = 2
\end{align*}
\]

\[A = \frac{1}{2} \| \overrightarrow{PQ} \| \| \overrightarrow{PR} \| \sin \theta = \frac{1}{2} \| \overrightarrow{PQ} \| \| \overrightarrow{PR} \| \sin \theta \]

(by (i))

(2) Find the area of the triangle formed by \( P, Q, \) and \( R \).

\[
h = \| \overrightarrow{PR} \| \sin \theta = \sqrt{5} \]

\[
b = \| \overrightarrow{PQ} \| = 1
\]

\[
A = \frac{1}{2}bh = \frac{1}{2} \| \overrightarrow{PQ} \| \| \overrightarrow{PR} \| \sin \theta = \frac{1}{2} \sqrt{5} \times \sqrt{5} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}
\]

(3) Find the angle between the planes defined by \( x + z = 1 \) and \( x - 2y - z = 2 \). (You can leave your answer in terms of an inverse trig function.)

\[
\vec{n}_1 = \text{normal vector for plane 1} = (1, 0, 1)
\]
\[
\vec{n}_2 = \text{normal vector for plane 2} = (1, -2, -1)
\]

\[
\vec{n}_1 \cdot \vec{n}_2 = \| \vec{n}_1 \| \| \vec{n}_2 \| \cos \theta \]

\[
\Rightarrow \cos \theta = \frac{1 \cdot 1 + 0 \cdot (-2) + 1 \cdot (-1)}{\sqrt{2} \sqrt{6}} = 0
\]

\[
\Rightarrow \theta = \frac{\pi}{2}
\]

(4) Write a Haiku in praise of the Cross Product.

You've crossed my product, 
For the last time, soul dot! 
Out, out, dread scalar!