Quiz 7

(1) Please fill in the following trigonometric identities, as discussed in relation to trigonometric substitution:

\[ 1 + \tan^2 x = \sec^2 x \]

\[ 1 - \sin^2 x = \cos^2 x \]

\[ \sec^2 x - 1 = \tan^2 x \]

(2) Please identify the appropriate trigonometric substitution for integrals involving the following forms:

<table>
<thead>
<tr>
<th>Form</th>
<th>Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^2 - x^2 )</td>
<td>( x = a \sin \theta )</td>
</tr>
<tr>
<td>( x^2 + a^2 )</td>
<td>( x = a \tan \theta )</td>
</tr>
<tr>
<td>( x^2 - a^2 )</td>
<td>( x = a \sec \theta )</td>
</tr>
</tbody>
</table>

(3) Find

\[ \int \frac{dy}{\sqrt{9 + y^2}} \]

\[ x = 3 \sec \theta \]

\[ dy = 3 \sec \theta \tan \theta d\theta \]

\[ \int \frac{3 \sec^2 \theta d\theta}{3 \sec \theta} = \int \sec \theta \tan \theta d\theta = \ln |\sec \theta + \tan \theta| + C \]

(4) Draw a triangle (and label the sides) that illustrates the relation \( x = 3 \tan \theta \).

(5) If \( x = 3 \tan \theta \), what is \( \sec \theta \)? (Hint: use the triangle in the last problem.)