Problem Statement: Assume \( f : S \rightarrow T \) is uniformly continuous on \( S \), where \( S \) and \( T \) are metric spaces. If \( \{x_n\} \) is any Cauchy sequence in \( S \), prove that \( \{f(x_n)\} \) is a Cauchy sequence in \( T \).

Proof. Since \( f \) is uniformly continuous, given \( \epsilon > 0 \), there exists \( \delta > 0 \) dependent only on \( \epsilon \) such that if \( d_S(x, y) < \delta \) then

\[
d_T(f(x), f(y)) < \epsilon
\]

for \( x, y \in S \). For this \( \delta \), since \( \{x_n\} \) is Cauchy, there exists an \( N \) such that

\[
d_S(x_n, x_m) < \delta
\]

whenever \( n, m \geq N \). Hence,

\[
d_T(f(x_n), f(x_m)) < \epsilon
\]

whenever \( n, m \geq N \), so \( \{f(x_n)\} \) is Cauchy. \( \Box \)