Problem Statement: Prove or disprove the following.

Let $F$ be a collection of sets in $\mathbb{R}^n$, and let $S = \bigcup_{A \in F} A$ and $T = \bigcap_{A \in F} A$.

1. If $x$ is an accumulation point of $T$ then $x$ is an accumulation point of each $A$ in $F$.

   **Proof.** Let $x$ be an accumulation point of $T$. Then for any ball centered at $x$, there is an element $t \neq x$ in $T$. Since $T$ is the intersection of all $A$ in $F$, $t$ is an element common to every set in $F$. This shows that if $x$ is an accumulation point of $T$, $x$ is an accumulation point for each $A$ in $F$. \qed

2. $x$ is an accumulation point of $S$, then $x$ is an accumulation point of at least one $A$ in $F$.

   A counterexample: Let $Q_n$ be the set of all fractions in $\mathbb{R}^1$ with denominator $n$ in lowest terms. For example $Q_6 = \{\ldots, \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \ldots\}$ Then $S = \bigcup_{n \in \mathbb{N}} Q_n = \mathbb{Q}$, and every real number is an accumulation point of $S$, but $\sqrt{2}$ for example is not an accumulation point for any single $Q_n$. 