Problem Statement: In any metric space \((M,d)\), prove that the empty set \(\emptyset\) and the whole space \(M\) are both open and closed

Proof. Let \(M\) be a metric space. For every point \(x_i \in M\) a ball \(B(x_i, r)\) with \(r > 0\) can be constructed around all such points within \(M\), thus \(M\) is open. The definition of a closed set is a set whose complement is an open set. Given the null set, \(\emptyset\), the complement of \(\emptyset\) is \(M\), and since \(M\) is open, \(\emptyset\) is closed. Conversely, if given the empty set, \(\emptyset\), a ball \(B(x_i, r)\) with \(r > 0\) can be constructed around every point since there are no points within \(\emptyset\). This means the empty set is open. Therefore since \(\emptyset\) is the complement of \(M\) and \(\emptyset\) is open, \(M\) must be closed. Thus both \(M\) and \(\emptyset\) and both open and closed. \(\square\)