Problem Statement: Consider the metric space \( \mathbb{Q} \) of rational numbers with the Euclidean metric of \( \mathbb{R} \). Let \( S \) consist of all rational numbers in the open interval \( (a, b) \), where \( a \) and \( b \) are irrational. Then \( S \) is closed and bounded subset of \( \mathbb{Q} \) which is not compact.

Proof. Suppose for contradiction that \( S \) is compact. Construct an open covering \( T = \{ B_r(x) | r = \frac{x-a}{2}, x \in S \} \). Then there exists no finite sub-cover of \( T \) since \( a, b \) are both irrational. Thus, \( S \) is not compact.

Next, note that \( S \) is bounded. Let \( x \in \mathbb{Q} - S \), then \( x < a \), or \( x > b \). If \( x < a \) then \( B_r(x) = (x-r, x+r) \cap \mathbb{Q} \subseteq \mathbb{Q} - S \), where \( d = d(x, a) \). Likewise, if \( x > b \) then \( B_r(x) = (x-r, x+r) \cap \mathbb{Q} \subseteq \mathbb{Q} - S \), where \( d = d(x, b) \). Hence, \( x \) is an interior point of \( \mathbb{Q} - S \). That is, \( \mathbb{Q} - S \) is open, or equivalently, \( S \) is closed.