Solution by Carly Osherow
Problem 1.20

**Problem Statement:** Prove the comparison property for suprema.

Proof. Given nonempty subsets $S$ and $T$ of $\mathbb{R}$ such that $s \leq t$ for every $s$ in $S$ and $t$ in $T$. If $T$ has a supremum then $S$ has a supremum and

$$\sup S \leq \sup T.$$ 

Let $t_0 \in T$. If $\sup T$ exists, we know $t_0 \leq \sup T$ and $s \leq t_0$, so $t_0$ is an upper bound for $S$, thus $S$ has a supremum by the completeness axiom. From this we gather $\sup S \leq t_0 \leq \sup T$, thus $\sup S \leq \sup T$. \qed