Solution by Caleb Spring
Problem 1.29

Note: The conjugate of \(z\) is \(\bar{z} = x - iy\) and \(|z| = \sqrt{x^2 + y^2}\) so \(|z|^2 = x^2 + y^2\)

**Problem Statement:** Prove that \(\bar{z}_1 + \bar{z}_2 = \bar{z}_1 + \bar{z}_2\)

a. The calculations are as follows,

\[
\bar{z}_1 + \bar{z}_2 = (x_1 + x_2) + i(y_1 + y_2)
= (x_1 + x_2) - i(y_1 + y_2)
= (x_1 - iy_1) + (x_2 - iy_2)
= \bar{z}_1 + \bar{z}_2 \quad \Box
\] (1)

**Problem Statement:** Prove that \(\bar{z}_1 \bar{z}_2 = \bar{z}_1 \bar{z}_2\)

b. The calculations are as follows,

\[
\bar{z}_1 \bar{z}_2 = (x_1 x_2 + i x_1 y_2 + i x_2 y_1 - y_1 y_2)
= (x_1 x_2 - y_1 y_2 - i x_1 y_2 + x_2 y_1)
= (x_1 - iy_1)(x_2 - iy_2)
= \bar{z}_1 \bar{z}_2 \quad \Box
\] (2)

**Problem Statement:** Prove that \(z\bar{z} = |z|^2\)

c. The calculations are as follows,

\[
z\bar{z} = (x + iy)(x + iy)
= (x + iy)(x - iy)
= xx - (i)(i)yy
= x^2 - (-1)y^2
= x^2 + y^2
= |z|^2 \quad \Box
\] (3)

**Problem Statement:** Prove that \(z + \bar{z} = 2x\), \((x\text{ is the real part})\)

d. The calculations are as follows,

\[
z + \bar{z} = (x + iy) + (x - iy)
= (x + x) + i(y - y)
= 2x + i(0)
= 2x \quad \Box
\] (4)

**Problem Statement:** Prove that \(t(z - \bar{z})/i = 2y\), \((y\text{ being the imaginary part})\)
e. The calculations are as follows,

\[
(z - \bar{z})/i = ((x + iy) - (\bar{x} + iy))/i \\
= ((x + iy) - (x - iy))/i \\
= ((x - x) + i(y + y)) \\
= i(y + y)/i \\
= 2y \square
\]