Apostol, Mathematical Analysis, 2nd ed.

Reading Guide:

Write your chapter notes in your notebook using some kind of navigation system to keep track of where in the book your notes pertain (e.g. page and section number, or page and paragraph number, or page and theorem number etc.)

These chapter notes MUST contain the items below as a subset, and you will be expected to discuss your reflections intelligently in class.

♦ indicates an item I will be specifically looking for when I review your notebook. Please write these items up especially carefully and make sure they are easy for me to find by marking the upper right hand corner of the page with a ♦ and a label such as ‘alternative proof of theorem 2.3’.

Chapter 2.

2.3 Read, but don’t obsess—we won’t really use this much. Your usual idea of an ordered pair is just fine.

2.4 The first example: Does this seem right to you? In what ways is RxR like C, and in what ways is it not like? The more advanced you are in math, the more skeptical you need to be in your reading. At this level math is only right until it is proved wrong—you need to start being suspicious that what you read is wrong. Read VERY critically.

2.5 For definitions 2.4 and 2.5: Is a relation always a function? If yes, prove it. If no, give a counter example. Is a function always a relation? If yes, prove it. If no, give a counter example.

What do these definitions of a function have to do with that vertical line test from Calc I/High School?

Which of Definition 2.5 and 2.6 is closest to the definition from Calc I and High School?

How do Definitions 2.5 and 2.6 compare to the definition of a function on Wikipedia (particularly the set theoretic definition)?

2.6 Check that if F(z)=z^2, then 0<=arg(z)<=alpha<= Pi/2  =>  0<=arg(F(z))<=alpha^2.

What does it mean for G contained in F if G and F are functions???? This doesn’t make sense with defn 2.6, which may be how you are used to thinking of functions, but does it make sense (and how) with defn 2.5?

27. Again this is a different (more general and more flexible) way of looking at something you already ‘know’. What does this have to do with the vertical and horizontal line tests from Calc I/HS? How is this definition of inverse more general than what you knew before? Give a couple examples for inverse functions using this where
the vertical line test is useless. An important thing to remember throughout this chapter is that the sets involved not only don’t have to be real numbers, but can be anything at all.

2.9 Go back to your Calc II book and review sequences. Think about your intuitive notion of a subsequence and how the formalism here makes it precise.

Make up two more examples.

2.10 Where have you seen equivalence relations before? Is ~ an equivalence relation of sets (see Exercise 2.2 if you have not seen the idea of an equivalence relation before)? Note—the terminology is lousy. The use of ‘relation’ in an equivalence relation here is focusing on a very different idea from the use of ‘relation’ in section 2.5.

Are RxR and C equivalent as sets? Prove it. Also prove the bit where it says “the proof is left to the reader”.

2.12 Give some examples of countably infinite sets and prove they are countable.

Theorem 2.16 is important to understand. Rewrite it for yourself.

2.13 This is a classic theorem, and most books draw a picture listing the s_n’s from say n=0 to 6, with … going to the right and down. Then look at the entrees on the diagonal. v_n,n is set up to disagree with all these diagonal entrees. Make this diagram in your notes, and we can discuss it in class if you wish.

Is Z^+ x Z^+ x Z^+ … x Z^+ countable for any (finite) number of Z^+’s? Prove it.

2.14 Give some concrete examples to go with Definitions 2.20 and 2.21.

Below Definition 2.21, it says that Definitions 2.20 and 2.21 apply even when F is not countable. Make up some examples for both union and intersection for uncountable sets.

2.15 For Definitions 2.24, I am likely to say “F is a collection of pairwise disjoint sets” instead of just ‘disjoint sets’.

Read theorem 2.25 and STOP—DO NOT READ AHEAD. Ask you self if the work ‘disjoint’ is really necessary for the theorem. Do you think the theorem would still be true if the word ‘disjoint’ were omitted? See if you can prove it before reading Theorems 2.26 and 2.27.

Theorem 2.27 is actually the important version, and the version we will mostly use. The author could easily have called Theorems 2.25 and 2.26 just Lemmas.

Come up with at least one more example like examples 1 and 2.

♦ Look back over this reading guide and the one for chapter 1. Notice that there are some common themes to the activities to do when reading mathematics closely. Based
on these, and the things you find useful for your self, come up with a list that will be a ‘generic reading guide’ for other chapters and other books. We can put this up in class and discuss if that would be useful to you.