Quiz 3

Each problem is worth 2 points. Please show your work. No calculators or technology allowed, but it is OK to leave work in relatively unsimplified form.

1. What is a quadratic surface? Write down the general equation that such a surface must satisfy. Be sure to specify which coefficients cannot be zero.

\[ A x^2 + B y^2 + C z^2 + D x y + E y z + F x z + G x + H y + I z + J = 0 \]

At least one of \( A \ldots F \) cannot be zero.

2. Consider the equation \( x^2 + 2y^2 = z^2 - 1 \). Sketch the graph of this surface. Where does the graph cross the \( z \)-axis? What do horizontal cross sections of the graph (i.e. intersections of the graph with planes of the form \( z = h \)) look like?

\[ \text{crosses } \varepsilon \text{-axis } \quad z = \pm 1 \]

\[ \text{intersection cut } \ell \text{ipses.} \]

3. Let \( P \) be the point whose rectangular coordinates are \( (0, 1, \sqrt{3}) \). Write \( P \) in both cylindrical and spherical coordinates.

\[ \text{cylindrical: } (r, \theta, z) = (1, \frac{\pi}{2}, \sqrt{3}) \]

\[ \text{spherical: } (\rho, \theta, \phi) = (2, \frac{\pi}{2}, \frac{\pi}{3}) \]
4. Consider the equation $r = 1$ in cylindrical coordinates. Sketch the graph of this equation.

\[ \text{infinite cylinder of radius 1} \]

\[ \text{oriented along z-axis} \]

5. Consider a sphere of radius 2 centered at the origin. Write down an equation in spherical coordinates that describes the portion of this ball that lies in the 1st octant (i.e. that describes the 1/8 of the ball situated along the positive $x$-, $y$-, and $z$-axes.)

\[ 0 \leq \rho \leq 2 \]
\[ 0 \leq \theta \leq \frac{\pi}{2} \]
\[ 0 \leq \phi \leq \frac{\pi}{2} \]