Chapter 12 Review

1. Find an $a$ such that the lines $r_1(t) = (1, 2, 1) + t(1, -1, 1)$ and $r_2 = (3, -1, 1) + t(a, 4, -2)$ intersect. (Challenge: is it obvious to you that such an $a$ should exist? Why?)

2. Let $v = \langle 1, 3, -2 \rangle$ and $w = \langle 2, -1, 4 \rangle$.
   (a) Compute $v \cdot w$

   (b) Compute the angle between $v$ and $w$.

   (c) Computer $v \times w$.

   (d) Find the area of the parallelogram spanned by $v$ and $w$.

   (e) Find the volume of the parallelepiped spanned by $v$, $w$, and $u = \langle 1, 2, 6 \rangle$. 
3. Let \( \mathbf{v} = \langle 1, -1, 3 \rangle \) and \( \mathbf{w} = \langle 4, -2, 1 \rangle \).

(a) Find the decomposition \( \mathbf{v} = \mathbf{v}_\parallel + \mathbf{v}_\perp \) with respect to \( \mathbf{w} \).

(b) Find the component of \( \mathbf{w} \) along \( \mathbf{v} \).

4. A 50-kg wagon is pulled to the right by a force \( F_1 \) making an angle 30° with the ground. At the same time the wagon is pulled to the left by a horizontal force \( F_2 \).

(a) Find the magnitude of \( F_1 \) in terms of the magnitude of \( F_2 \) if the wagon does not move.

(b) What is the maximal magnitude of \( F_1 \) that can be applied to the wagon without lifting it? (Hint: find a physicist to explain how much downward force a 50-kg wagon exerts.)

5. Show that if \( \mathbf{v} \) and \( \mathbf{w} \) are orthogonal, then \( \| \mathbf{v} + \mathbf{w} \|^2 = \| \mathbf{v} \|^2 + \| \mathbf{w} \|^2 \).
6. Write the equation of the plane \( \mathcal{P} \) with the vector equation

\[
\langle 1, 4, -3 \rangle \cdot \langle x, y, z \rangle = 7
\]

in the form

\[
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.
\]

7. Find the trace of the plane \( 3x - 2y + 5z = 4 \) in the \( xy \)-plane.

8. Determine the type of the quadratic surface \( ax^2 + by^2 - z^2 = 1 \) if

(a) \( a < 0, b < 0. \)

(b) \( a > 0, b > 0. \)

(c) \( a > 0, b < 0. \)
9. Sketch the graph of the cylindrical equation $z = 2r \cos \theta$, and write the equation in rectangular coordinates.

10. Describe the set of all points $P = (x, y, z)$ satisfying $x^2 + y^2 \leq 4$ in both cylindrical and spherical coordinates.