Section 14.1: Functions of More Than One Variable

Key points:

1. This section deals with scalar valued vector functions, i.e. functions \( f : \mathbb{R}^n \to \mathbb{R} \), where \( n \geq 2 \). (This notation means the function takes an \( n \)-dimensional vector as input, and gives a scalar as output.)

2. When the domain is a set in \( \mathbb{R}^2 \), the graphs of such functions are relatively easy to visualize; otherwise, they are generally not.

3. A big part of this chapter is about contour maps, which most of you will already know something about from having read topographical maps at some point in your life. Key features of how to read a contour map:
   - lines represent paths of “constant elevation” (they are called level lines.)
   - the change in elevation between adjacent lines is fixed
   - the steepest ascent (and descent) directions are perpendicular to the level lines.
   - the average rate of change between two points \( P \) and \( Q \) is \( \frac{\Delta \text{altitude}}{\Delta \text{horizontal}} \).

Problems:

1. Draw a contour map of the function \( f(x, y) = x + y \)

2. Suppose you are told that the topography of a certain region satisfies the equation \( f(x, y) = x^2 y^2 \), and you travel from \( P = (0, 1) \) to \( Q = (2, 3) \). What was your average rate of change?

3. Describe the vertical and horizontal traces of \( f(x, y) = y^2 \).

4. Describe what a level surface of the function \( f(x, y, z) = x^2 + y^2 + z^2 \) looks like.