General protocol for writing up homework solutions:

- Solutions should be written in complete, grammatically correct sentences.
- Solutions should illustrate a thought process, not merely provide an answer.
- Most of these solutions will be discussed in class on the day the homework is due—come prepared to present or discuss.

Textbook problems:

- Pg. 15, Exercise 1.1
- Pg. 16, Exercise 1.2
- Pg. 16, Exercise 1.3

Additional problems:

1. In class we started with the single trapezoidal rule,

\[ \int_a^b f(x)dx \approx b - a \cdot \frac{f(b) + f(a)}{2} \]  

(single trapezoid rule)

and used this formula to derive the composite trapezoid rule,

\[ \int_a^b f(x)dx \approx \frac{b - a}{n} \left[ \frac{f(a)}{2} + \frac{f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right] \]  

(composite trapezoid rule)

where \( n \) is the number of trapezoids and \( x_i = a + i(b - a)/n \). Derive a composite midpoint rule, where the single midpoint approximation is given by

\[ \int_a^b f(x)dx \approx (b - a)f \left( \frac{a + b}{2} \right) \]  

(single midpoint rule).

2. This exercise tests your understanding of Taylor series.

(a) Find the second-order Taylor series of the function \( f(x) = \cos(x) \) at the point \( x_0 = \pi \). Be sure to include an error term.

(b) Use your solution to part a.) to approximate the value of \( \cos(5\pi/4) \).

(c) Use the error term from part a.) to provide an upper bound on your approximation in b.)

(d) Calculate the actual error in your estimate, and verify that it satisfies the bound specified in c.)