1. Suppose \( f(x) = \frac{x^2}{x-3} \). Use the linearization at the point \( x = 4 \) to estimate \( f(4.1) \).

**Solution:**

Since \( f'(x) = \frac{x^2 - 6x}{(x-3)^2} \), it follows that \( f'(4) = -8 \). Since \( f(4) = 16 \), the tangent line is given by

\[ L(x) = -8(x-4) + 16. \]

Plugging in \( x = 4.1 \) yields \( L(4.1) = 15.2 \), which is our estimate for \( f(4.1) \).

2. Find the slope of the tangent at the point \( P = (1,1) \) on the graph of \( e^{x-y} = 2x^2 - y^2 \).

**Solution:**

Applying the operator \( d/dx \) to both sides of the equation yields:

\[ e^{x-y} \cdot (1 - y') = 4x - 2yy'. \]

Substituting \((1,1)\) for \((x,y)\) yields

\[ 1 - y' = 4 - 2y', \]

i.e. \( y' = 3 \).

3. Find all the critical points of the function \( g(x) = \frac{1}{x-1} - \frac{1}{x} \).

**Solution:**

The domain of this function is the set of all real numbers, except 1 and 0. The derivative is given by

\[ g'(x) = \frac{-1}{(x-1)^2} + \frac{1}{x^2}, \]

which is also undefined at 1 and 0. Since these points don’t lie in the domain, however, we don’t call them critical points. Since \( g'(1/2) = -1/4 + 1/4 = 0 \), the point \( x = 1/2 \) is a critical point.

4. Find the maximum and the minimum of the function \( f(x) = 2\sqrt{x^2+1} - x \) on the interval \([0, 2]\).

**Solution:**

Note that \( f'(x) = \frac{2x}{\sqrt{x^2+1}} - 1 \), and that this is equal to zero at \( x = 1/\sqrt{3} \). Candidates for max’s and min’s thus consist of the endpoints and this critical point. Comparing the value of \( f \) at each of these points, we see that there is a minimum at \( 1/\sqrt{3} \) and a maximum at 2.