Limit Practice Problems

Math 180 -- 8/11/15 -- Toews

Definition of a limit:
Suppose a function \( f(x) \) has the following values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>9.6100</td>
</tr>
<tr>
<td>2.9</td>
<td>8.4100</td>
</tr>
<tr>
<td>3.01</td>
<td>9.0601</td>
</tr>
<tr>
<td>2.99</td>
<td>8.9401</td>
</tr>
<tr>
<td>3.001</td>
<td>9.0060</td>
</tr>
<tr>
<td>2.999</td>
<td>8.9940</td>
</tr>
</tbody>
</table>

What is the limit of \( f(x) \) as \( x \) goes to 3?

Answer: 9

Left and right sided limits:
What are \( \lim_{x \to 3^-} f(x) \) and \( \lim_{x \to 3^+} f(x) \)?

If the graph of \( f(x) \) is:

![Graph](image)

What is the limit of \( f(x) \) as \( x \) goes to 3?
Left and right sided limits:

What are
\[ \lim_{x \to 3^+} f(x) \text{ and } \lim_{x \to 3^-} f(x) \]
if the graph of \( f(x) \) is:

Answer: both are equal to 1.

More one sided limits:

What are
\[ \lim_{x \to 3^+} f(x) \text{ and } \lim_{x \to 3^-} f(x) \]
if the graph of \( f(x) \) is:

Answer: 2 and 3, respectively.

Infinite Limits:

Venture a guess as to what the following limit is:

\[ \lim_{x \to 3} \frac{1}{(x - 3)^2} \]
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\[
\lim_{x \to 3} \frac{1}{(x - 3)^2}
\]
Answer: infinity

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Infinite Limits:
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\]
Answer: does not exist (write DNE). But one sided limits do exist:
\[
\lim_{x \to 3^-} f(x) = \infty \quad \text{and} \quad \lim_{x \to 3^+} f(x) = -\infty
\]

Average Velocity
Suppose the position of a particle is given by
\[
s(t) = 3t^2
\]
Find the average velocity in the interval [4, 4.1]
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Suppose the position of a particle is given by
\[ s(t) = 3t^2 \]
Find the average velocity in the interval \([4, 4.1]\).

Answer: 24.3. Exact!

Instantaneous Velocity
Suppose the position of a particle is given by
\[ s(t) = 3t^2 \]
Find the instantaneous velocity at time \(t=4\).

Instantaneous Velocity
Suppose the position of a particle is given by
\[ s(t) = 3t^2 + 2 \]
Find the instantaneous velocity at \(t=4\). Is your answer a guess, or are you 100% sure of it?
Instantaneous Velocity

Suppose the position of a particle is given by

\[ s(t) = 3t + 2 \]

Find the instantaneous velocity at \( t=4 \). Is your answer a guess, or are you 100% sure of it?

Answer: \( v(4) = 3 \).

100% sure! Since \( s(t) \) is linear, the secant lines all have the same slope!

Limit Laws—Teaser Problem

Suppose

\[ \lim_{x \to 2} f(x) = 1 \quad \text{and} \quad \lim_{x \to 2} g(x) = 3. \]

What is

\[ \lim_{x \to 2} [f(x) + g(x)] \]

Defend your answer!

Answer:

\[ \lim_{x \to 2} [f(x) + g(x)] = 4 \]

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Answer:

\[ \lim_{x \to 2} [f(x) + g(x)] = 4 \]
Limit Laws—more practice

Evaluate the following:

• (2.3, #2) \( \lim_{x \to 3} 14 \)
  
  Answer: 14

• (2.3, #4) \( \lim_{z \to 27} z^{2/3} \)
  
  Answer: 9

• (2.3, #24) \( \lim_{z \to 7} \frac{(t + 2)^{1/2}}{(t + 1)^{2/3}} \)
  
  Answer: 3/4
Important points:

1. Two ways to figure out limits:
   • Make a table of values
   • Look at a graph

2. The value of \( \lim_{x \to c} f(x) \) does not depend on \( f(c) \).

3. Left and right sided limits must be the same in order to say that “the limit” exists

4. Limits can be infinity or negative infinity

5. Instantaneous velocity is defined as a limit
   • It is the limit of average velocities over smaller and smaller intervals