Exam 3

Please show your work. Each subproblem is worth 5 points.

Problem 1  (50 pts) Find $dy/dx$ for each of the following, using whatever technique you need.

a) $y = (x + 1)^{100}$

b) $y = \cos 5x$

c) $y = e^{3x}(x^2 + 1)$

d) $y = \frac{x}{x^3 + 2}$

e) $y = \cos^{-1}(2x)$
f) \[ y = 7^x + \log_7 x \]

g) \[ xy = y^2 + x + y \]

h) Let \( y(x) \) be the inverse of \( x^2 + 2x + 1 \). Find \( dy/dx \) at \( x = 4 \).

i) \[ y = \ln(\ln x) \]

j) \[ y = e^{f(x)g(x)} \] (leave your answer in terms of \( f, g \), and their derivatives.)
Problem 2  Consider the equation \( xy + x^2y^2 = 2 \).

a)  Find \( dy/dx \) at the point \((1,1)\).

b)  Write down the equation of the tangent line at the point \((1,1)\).

Problem 3  Consider the function \( f(x) = \sqrt{x} \).

a)  Write down the linearization \( L(x) \) of this function at the point \( x = 25 \).

b)  Use your linearization to estimate \( f(26) \).

c)  Which is larger, \( \sqrt{3.1} - \sqrt{3} \) or \( \sqrt{7.1} - \sqrt{7} \)? Explain using the idea of a linear approximation.
Problem 4  Consider the function \( f(x) = x^3 - 6x^2 + 8 \)

a) Find all the critical points of this function in the interval \([-2, 1]\)

b) Find the maximum and minimum of \( f(x) \) over this interval.

Problem 5  Recall that the position of an object in a gravitational field has the form \( x(t) = x_0 + v_0 t - 4.9t^2 \).
Suppose that a ball is thrown vertically upward with an initial velocity of 9.8 meters per second from an initial height of 0 meters.

a) Find the height of the ball at \( t = 2 \).

b) What is the ball’s maximum height, and when does it reach that height?

Problem 6  Let \( f(x) = xe^x \). Find \( f''(x) \).