Solutions

3.3.10 Let \( w(z) = \frac{z^2}{\sqrt{z} + z} \). Then
\[
\frac{dw}{dz} = \frac{(\sqrt{z} + z) \frac{d}{dz} z^2 - z^2 \frac{d}{dz} (\sqrt{z} + z)}{(\sqrt{z} + z)^2} = \frac{2z(\sqrt{z} + z) - z^2((1/2)z^{-1/2} + 1)}{(\sqrt{z} + z)^2} = \frac{(3/2)z^{3/2} + z^2}{(\sqrt{z} + z)^2}.
\]
Therefore,
\[
\left. \frac{dw}{dz} \right|_{z=9} = \frac{(3/2)(9)^{3/2} + 9^2}{(\sqrt{9} + 9)^2} = \frac{27}{32}.
\]

3.3.14 Let \( f(x) = x^2(3 + x^{-1}) \). Then, using the product rule, and then power and sum rules,
\[
f'(x) = x^2(-x^{-2}) + (3 + x^{-1})(2x) = 6x + 1.
\]
Multiplying out first, we find \( f(x) = 3x^2 + x \). Then \( f'(x) = 6x + 1 \).

3.3.38 Let \( f(x) = \left( \frac{ax + b}{cx + d} \right) \). Using the quotient rule:
\[
f'(x) = \frac{(cx + d)a - (ax + b)c}{(cx + d)^2} = \frac{(ad - bc)}{(cx + d)^2}.
\]

3.3.40 Let \( F(x) = x^2f(x) \). Then \( F'(x) = x^2f'(x) + 2xf(x) \), and
\[
F'(4) = 16f'(4) + 8f(4) = (16)(-2) + (8)(10) = 48.
\]

3.3.42 Let \( H(x) = \frac{x}{g(x)f(x)} \). Then
\[
H'(x) = \frac{g(x)f(x) \cdot 1 - x(g(x)f'(x) + f(x)g'(x))}{(g(x)f(x))^2},
\]
and
\[
H'(4) = \frac{(5)(10) - 4((5)(-2) + (10)(-1))}{((5)(10))^2} = \frac{13}{250}.
\]

3.4.2 Let the volume be \( V = f(s) = s^3 \). Then the rate of change of \( V \) with respect to \( s \) is \( \frac{d}{ds} s^3 = 3s^2 \). When \( s = 5 \), the volume changes at a rate of \( 3(5^2) = 75 \) cubic units per unit increase.

3.4.28 Suppose \( H \) is the unknown height from which the bucket fell starting at time \( t = 0 \). The height of the bucket at time \( t \) is \( s(t) = H - 4.9t^2 \). Let \( T \) be the time when the bucket hits the ground (thus \( S(T) = 0 \)). Olivia saw the bucket at time \( T - 1.5 \). The window is located 9.5 floors or 47.5 m above ground. So we have the equations
\[
s(T - 1.5) = H - 4.9(T - 1.5)^2 = 47.5 \quad \text{and} \quad s(T) = H - 4.9T^2 = 0
\]
Subtracting the second equation from the first, we obtain \(-4.9(-3T + 2.25) = 47.5\), so \( T \approx 4 \) s. The second equation gives us \( H = 4.9T^2 = 4.9(4)^2 \approx 78.4 \) m. Since there are 5 m in a floor, the bucket was dropped 78.4/5 \approx 15.7 \) floors above the ground. The bucket was dropped from the top of the 15th floor.
3.4.34 The minute hand makes one full revolution every 60 minutes, so the minute hand moves at a rate of
\[
\frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/min.}
\]
The hour hand makes one-twelfth of a revolution every 60 minutes, so the hour hand moves with a rate of
\[
\frac{\pi}{360} \text{ rad/min.}
\]
At 3 o’clock, the movement of the minute hand works to decrease the angle between the minute and hour hands while the movement of the hour hand works to increase the angle. Therefore, at 3 o’clock,
\[
\theta'(t) = \frac{\pi}{360} - \frac{\pi}{30} = -\frac{11\pi}{360} \text{ rad/min.}
\]

3.4.38 The position graph appears to break into four equal-sized components. Over the first quarter of the time interval, the position graph is rising but bending downward, eventually reaching a horizontal tangent. Thus, over the first quarter of the time interval, the velocity is positive but decreasing, eventually reaching 0. Continuing to examine the structure of the position graph produces the following graph of velocity:

![Velocity Graph](image)

3.5.4 Let \( y = 4t^3 - 9t^2 + 7 \). Then \( y' = 12t^2 - 18t, \) \( y'' = 24t - 18, \) and \( y''' = 24. \)

3.5.14 Let \( y = \frac{1}{\sqrt{x}}. \) Applying the quotient rule:
\[
y' = \frac{(1-x)(0) - 1(-1)}{(1-x)^2} = \frac{1}{1-2x+x^2}
\]
\[
y'' = \frac{(1-2x+x^2)(2) - (1)(-2+2x)}{(1-2x+x^2)^2} = \frac{2-2x}{1-x}
\]
\[
y''' = \frac{(1-3x+3x^2-x^3)(0) - 2(-3+6x-3x^2)}{(1-3x+3x^2-x^3)^2} = \frac{6(x^2-2x+1)}{(1-x)^4}
\]

3.5.32 Let \( f(x) = (x+2)^{-1} = \frac{1}{x+2}. \) Then \( f'(x) = -(x+2)^{-2}, \) \( f''(x) = 2(x+2)^{-3}, f'''(x) = -6(x+2)^{-4}, \)
\( f^{(4)}(x) = 24(x+2)^{-5}, \ldots \) From this we conclude that the \( n \)th derivative can be written as
\[
f^{(n)}(x) = (-1)^n n!(x+2)^{-(n+1)}.
\]

3.5.40 \( f'(x) = A \) and \( f(x) = B. \)

3.6.2 Let \( f(x) = \cos x. \) Then \( f'(x) = -\sin x \) and the equation of the tangent line is
\[
y = f'(\frac{\pi}{3})(x - \frac{\pi}{3}) + f(\frac{\pi}{3}) = -\sqrt{3} \left( x - \frac{\pi}{3} \right) + \frac{1}{2} = -\frac{\sqrt{3}}{2} x + \frac{1}{2} + \frac{\sqrt{3}}{6}.
\]

3.6.8 Let \( f(x) = 9 \sec x + 12 \cot x. \) Then \( f'(x) = 9 \sec x \tan x - 12 \csc^2 x. \)

3.6.14 Let \( f(z) = z \tan z. \) Then \( f'(z) = z(\sec^2 z) + \tan z. \)

3.6.22 Let \( h(t) = e^t \csc t. \) Then \( h'(t) = e^t(-\csc t \cot t) + \csc t e^t = e^t \csc t(1-\cot t). \)

3.6.26 Let \( f(\theta) = \tan \theta. \) Then \( f'(\theta) = \sec^2 \theta \) and \( f'(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}. \) The tangent line at \( x = \frac{\pi}{6} \) is
\[
y = f'(\frac{\pi}{6})(\theta - \frac{\pi}{6}) + f\left(\frac{\pi}{6}\right) = \frac{4}{3} \left( \theta - \frac{\pi}{6} \right) + \frac{\sqrt{3}}{3} = \frac{4\theta}{3} + \frac{\sqrt{3}}{3} - \frac{2\pi}{9}.
\]

3.6.40
\[
\frac{d}{dt} \cos^2 t = \frac{d}{dt}(\cos t \cdot \cos t) = \cos(t(-\sin t) + \cos(-\sin t)) = -2\sin t \cos t
\]
\[
\frac{d^2}{dt^2} \cos^2 t = \frac{d}{dt}(-2\sin t \cos t) = -2(\sin(t(-\sin t) + \cos t)) = -2(\cos^2 t - \sin^2 t).
\]