2.4.4 The function \( g \) has a jump discontinuity at \( x = 1 \), but is left-continuous there. Assigning \( g(1) = 3 \) makes \( g \) right-continuous at \( x = 1 \) (but no longer left-continuous).

2.4.20 This function has a jump discontinuity at \( x = n \) for every integer \( n \). It is continuous at all other values of \( x \). For every integer \( n \),
\[
\lim_{x \to n^+} [x] = n
\]
since \([x] = n\) for all \( x \) between \( n \) and \( n + 1 \). This shows that \([x]\) is right-continuous at \( x = n \). On the other hand,
\[
\lim_{x \to n^-} [x] = n - 1
\]
since \([x] = n - 1\) for all \( x \) between \( n - 1 \) and \( n \). Thus \([x]\) is not left-continuous.

2.4.22 The function \( f(t) = \frac{1}{t^2 - 1} = \frac{1}{(t-1)(t+1)} \) is discontinuous at \( t = -1 \) and \( t = 1 \), at which there are infinite discontinuities. The function is neither left- nor right-continuous at either point of discontinuity.

2.4.28 The function \( \cos \left( \frac{1}{x} \right) \) is discontinuous at \( x = 0 \), at which there is an oscillatory discontinuity. Because neither
\[
\lim_{x \to 0^-} f(x) \text{ nor } \lim_{x \to 0^+} f(x)
\]
exist, the function is neither left- nor right-continuous at \( x = 0 \).

2.4.52 The function \( f \) is right-continuous at \( x = 1 \).

2.4.58 As \( x \to 3^- \), we have \( 2x + 9x^{-1} \to 9 = L \). As \( x \to 3^+ \), we have \( -4x + c \to c - 12 = R \). Match the limits: \( L = R \) or \( 9 = c - 12 \) implies \( c = 21 \).

2.4.62 Refer to the four figures shown below.

(a) The figure at the top left shows a function for which \( \lim_{x \to a} f(x) \) exists, but the function is not continuous at \( x = a \) because the function is not defined at \( x = a \).

(b) The figure at the top right shows a function that has a jump discontinuity at \( x = a \) but \( f(a) \) is not equal to either \( \lim_{x \to a^-} f(x) \) or \( \lim_{x \to a^+} f(x) \).

(c) This statement can be false either when the two one-sided limits exist and are equal or when one or both of the one-sided limits does not exist. The figure at the top left shows a function that has a discontinuity at \( x = a \) with both one-sided limits being equal; the figure at the bottom left shows a function that has a discontinuity at \( x = a \) with a one-sided limit that does not exist.

(d) The figure at the bottom left shows a function for which \( \lim_{x \to a} f(x) \) does not exist and one of the one-sided limits also does not exist; the figure at the bottom right shows a function for which \( \lim_{x \to a} f(x) \) does not exist and neither of the one-sided limits exists.
2.4.64

2.5.8 \( \lim_{x \to 8} \frac{x^3 - 64x}{x - 8} = \lim_{x \to 8} \frac{x(x - 8)(x + 8)}{x - 8} = \lim_{x \to 8} x(x + 8) = 8(16) = 128. \)

2.5.14

\[
\lim_{h \to 0} \frac{(3 + h)^3 - 27}{h} = \lim_{h \to 0} \frac{27 + 27h + 9h^2 + h^3 - 27}{h} = \lim_{h \to 0} \frac{27h + 9h^2 + h^3}{h} = \lim_{h \to 0} (27 + 9h + h^2) = 27 + 9(0) + 0^2 = 27.
\]

2.5.26

\[
\lim_{h \to 0} \frac{(3 + h)^2 - 9a^2}{h} = \lim_{h \to 0} \frac{6ah + h^2}{h} = \lim_{h \to 0} (6a + h) = 6a.
\]

2.5.48

Because \( \lim_{x \to 0} \cos x = \lim_{x \to 0} 1 = 1 \), it follows that \( \lim_{x \to 0} f(x) = 1 \) by the Squeeze Theorem.

2.6.4 Multiplying the inequality \( |\sin \frac{1}{x^2}| \leq 1 \), which holds for \( x \neq 0 \), by \( |x| \) yields \( |x\sin \frac{1}{x^2}| \leq |x| \) or \( -|x| \leq x \sin \frac{1}{x^2} \leq |x| \). Because

\[
\lim_{x \to 0} -|x| = \lim_{x \to 0} |x| = 0,
\]

it follows by the Squeeze Theorem that

\[
\lim_{x \to 0} x \sin \frac{1}{x^2} = 0.
\]

2.6.18

\[
\lim_{x \to 0} \frac{\sin x \sec x}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \sec x = 1 \cdot 1 = 1.
\]

2.6.24

\[
\lim_{\theta \to 0} \frac{1 - \cos \theta}{\sin \theta} = \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} \cdot \lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 0 \cdot 1 = 0.
\]

2.6.30 Let \( x = 4h \). Then \( x \to 0 \) as \( h \to 0 \) and

\[
\lim_{h \to 0} \frac{\sin 4h}{4h} = \lim_{x \to 0} \frac{\sin x}{x} = 1.
\]

2.7.6 (a) From the tables below, it appears that

\[
\begin{array}{c|cccc}
\hline
x & -100 & -500 & -1000 & -10000 \\
\hline
f(x) & -5.994326 & -5.998973 & -5.999493 & -5.999950 \\
\hline
\end{array}
\]

2.5.8

\( \lim_{x \to 8} \frac{x^3 - 64x}{x - 8} = \lim_{x \to 8} \frac{x(x - 8)(x + 8)}{x - 8} = \lim_{x \to 8} x(x + 8) = 8(16) = 128. \)
\begin{equation}
\begin{array}{c|ccccc}
  x & 100 & 500 & 1000 & 10000 \\
  \hline
  f(x) & 6.004325 & 6.000973 & 6.000493 & 6.000050 \\
\end{array}
\end{equation}

(b) From the graph below, it also appears that
\[
\lim_{x \to \infty} \frac{12x + 1}{\sqrt{4x^2 + 9}} = 6 \quad \text{and} \quad \lim_{x \to -\infty} \frac{12x + 1}{\sqrt{4x^2 + 9}} = -6.
\]

(c) The horizontal asymptotes of \( f(x) \) are \( y = -6 \) and \( y = 6 \).

2.7.18 First calculate the limits as \( x \to \pm \infty \). For \( x \to \infty \),
\[
\lim_{x \to \infty} \frac{8x^3 - x^2}{7 + 11x - 4x^4} = \lim_{x \to \infty} \frac{8x - 1}{x + \frac{11}{x} - 4} = 0.
\]
Similarly,
\[
\lim_{x \to -\infty} \frac{8x^3 - x^2}{7 + 11x - 4x^4} = \lim_{x \to -\infty} \frac{8x - 1}{x + \frac{11}{x} - 4} = 0.
\]
Thus, the horizontal asymptote of \( f(x) \) is \( y = 0 \).

2.7.28 \[ \lim_{t \to \infty} \frac{t^{4/3} - 9t^{1/3}}{(8t^4 + 2)^{1/3}} = \lim_{t \to \infty} \frac{1 - \frac{9}{7}}{(8 + \frac{2}{t^4})^{1/3}} = \frac{1}{2}. \]

2.7.30 Because \[ \lim_{t \to -\infty} e^{2t} = \lim_{t \to -\infty} e^{3t} = 0, \]
it follows that \[ \lim_{t \to -\infty} \frac{4 + 6e^{2t}}{5 - 9e^{3t}} = \frac{4 + 0}{5 - 0} = \frac{4}{5}. \]