Remember that the purpose of the exam is to show me how you think about these problems. Accordingly, please show your work, use mathematical notation correctly, and justify your answers.

1. **(20 points) Practice using tables.** Attached to the back of this exam are a binomial table, a $T$–table, and a standard normal table. Use these tables to answer the following questions:
   
   (a) If $X$ is distributed binomially with parameters $n = 6$ and $p = .20$, calculate the probability that $X ≥ 2$.

   (b) Find $z^*$ such that the area under a $N(0, 1)$ curve between $-z^*$ and $z^*$ is 0.99.

   (c) Suppose $X$ is distributed $N(0, 1)$. Find the probability that $X$ is bigger than 1.23.

   (d) Find $t^*$ such that 90% of the area under a $T(15)$ curve is between $-t^*$ and $t^*$.

   (e) Suppose $X$ is distributed $T(15)$. Estimate the probability that $X$ is bigger than 1.5.

2. **Binomial random variables.** It is a fact that one in ten of your fellow students is really an alien. Suppose you take an SRS of size 100 from the population of your fellow students, and let $X$ be the total number of aliens in your sample.

   (a) **(6 points)** Explain why $X$ is distributed binomially, and give values for the parameters $n$ and $p$. 
(b) **(5 points)** What are the mean and standard deviation of $X$?

(c) **(4 points)** Is it OK to use a normal distribution to approximate the distribution of $X$? **Justify** your answer.

(d) **(5 points)** Calculate the approximate probability that $X \leq 15$.

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3. **Margins of error.** You are interested in estimating the average peak daily temperature in your region. You plan to randomly choose some days in summer, record the peak temperature on these days, and report a 95% confidence interval for the mean. You know from historical data that the standard deviation is $\sigma = 6$ degrees.

(a) **(3 points)** Recall that the formula for margin of error is $m = z^* \sigma / \sqrt{n}$. What is $z^*$ in this case?

(b) **(3 points)** How big is your margin of error if your sample size is $n = 9$?

(c) **(3 points)** How big does your sample have to be in order to have a margin of error of less than 1°?
4. Suppose you wish to estimate the mean weight of a population of red pandas. You take an SRS of 16 pandas, and calculate the mean weight as \( \bar{x} = 325 \) pounds. You happen to know that the standard deviation of panda weights is \( \sigma = 8 \).

(a) (7 points) Compute a 90% confidence interval for the value of the population mean.

(b) (4 points) Would your margin of error get bigger or smaller if instead of reporting a 90% confidence interval, you reported a 95% confidence interval? Justify your answer.

(c) (4 points) Would your margin of error get bigger or smaller if instead of weighing 16 pandas, you weighed 25? Justify your answer.

(d) (6 points) Suppose one of your goals in this study was to see if red tailed pandas were getting fatter. Based on a previous study, your null hypothesis is of the form \( H_0 : \mu = 320 \), and your alternative is \( H_a : \mu > 320 \). What is the \( P \)-value of your data?

(e) (4 points) Can you reject the null hypothesis at the 5% level? Why or why not?
5. Suppose you are interested in testing whether a certain chemical additive influences a motorcycle’s top speed. You give 16 motorcycle riders a sample of your chemical additive and 25 motorcycle riders a “placebo” (something that looks and smells like the additive but has no effect.) The average top speed for people that got the additive was 118mph, while the average top speed for people that got the “placebo” was 120.

(a) (5 points) State an appropriate null hypothesis. (Define any symbols you introduce.)

(b) (5 points) State an appropriate alternative hypothesis. Justify this choice of alternative.

(c) (6 points) Suppose the sample standard deviations of the two groups are $s = 2$ for the group that got the additive and $s = 3$ for the group that got the “placebo”. Calculate the $t$–statistic for your data.

(d) (7 points) Use your $t$–statistic to give a $P$–value (with respect to the null and alternative listed above.) Can you reject the null hypothesis at the 1% level of significance?