Estimating the mean when you know $\sigma$

Formulas

1. The **margin of error** for a level $C$ confidence interval is

$$m = z^* \frac{\sigma}{\sqrt{n}},$$

where $\sigma$ is the standard deviation of the population, $n$ is the size of the SRS, and $z^*$ is the $Z$–score from a $N(0,1)$ table with area $C$ between $-z^*$ and $z^*$.

2. To find $z^*$ in the above formula, first find $-z^*$, which is given by

$$-z^* = Z – \text{score corresponding to } \frac{1 - C}{2}.$$

3. The **level $C$ confidence interval** for $\mu$ is

$$\bar{x} \pm m$$

where $m$ is as above.

4. If you wish to have a margin of error of size $m$ and a confidence level of size $C$, then you need an SRS of size

$$n = \left( \frac{z^* \sigma}{m} \right)^2.$$

Practice Problems

1. Suppose you don’t know how much you spend on coffee each day, but guess that the standard deviation for such expenses is $1.50. Suppose your take a random sample of 25 days, and let $\bar{x}$ be the average amount you spend on those days. What is the standard deviation of $\bar{x}$?

2. The 68-95-99.7 Rule states that the probability is about 95% that $\bar{x}$ is within $\$_$ of the population mean. Fill in the blank.

3. Suppose an SRS of 36 professional athletes was taken to measure their standing heart rates. The average standing heart rate in this sample was 54 beats per minute. If the standard deviation of standing heart rates is known to be 3, give a 95% confidence interval for $\mu$, the average standing heartrate for all professional athletes.
4. Repeat the above problem, but instead of giving a 95% confidence interval, give a 98% confidence interval. How has your answer changed from the last one? Does this change seem reasonable? Generalize your observation in a statement of the form “the higher the level of confidence, the __________ the longer the confidence interval.”

5. Now repeat the last two problems under the assumption that an SRS of size 64 was taken. How does this change your answers? Do these changes seem reasonable? Generalize your observations in a statement of the form “for a given confidence level, the larger the sample size, the __________ the confidence interval.”

6. Suppose you are designing a survey of sleeping habits of college freshmen. You don’t know the mean number of hours that freshmen sleep each night, but you suspect that the standard deviation is about .75. You plan to take an SRS of college freshmen and use the sample mean as an estimate of the population mean. In order to publish your findings, you need to publish a confidence interval along with your estimate: good studies usually have small margins of error and high confidence. If you wish to have a margin of error of 10 minutes and a 99% confidence, how many students should you sample?