Estimating the mean when you don’t know $\sigma$

Formulas

Suppose a simple random sample of size $n$ is taken from a normal population with unknown mean and standard deviation.

- A $C$ level confidence interval for $\mu$ is
  \[
  \bar{x} \pm t^* \frac{s}{\sqrt{n}},
  \]
  where $\bar{x}$ is the sample mean, $s$ is the sample standard deviation, and $t^*$ is the value such that area $C$ is between $-t^*$ and $t^*$ on a $T(n-1)$ curve. (The value $t^*s/\sqrt{n}$ is called the margin of error.)

- To calculate the $P-$value of $\bar{x}$ for a null hypothesis of the form $H_0 : \mu = \mu_0$, first form the test statistic
  \[
  t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.
  \]

Then

- if $H_a : \mu < \mu_0$, the $P-$value is the area under a $T(n-1)$ curve to the left of $t$.
- if $H_a : \mu > \mu_0$, the $P-$value is the area under a $T(n-1)$ curve to the right of $t$.
- if $H_a : \mu \neq \mu_0$, the $P-$value is the area under a $T(n-1)$ curve to the right of $|t|$ and to the left of $-|t|$.

Practice Problem

1. Suppose you take an SRS of size 5 and your data is as follows:
   
   $-1.64, 1.48, 0.17, -1.17, -0.82$.

   (a) Find $\bar{x}$.

   (b) Find $s$. 
(c) Estimate the mean of the population from which this data came from, and give 95% confidence bound for your estimate. What assumptions are you making?

(d) Use your data to test the hypothesis $H_0 : \mu = 0$ against the alternative $H_a : \mu \neq 0$. Can you reject $H_0$ at the 5% level?