1. Modeling Problem #7, pg. 17
2. Modeling Problem #8, pg. 17
3. Modeling Problem #9, pg. 17
4. Book Problem #4.1, pg. 58
5. (Practice with Matrix norms) The norm of a matrix $A \in \mathbb{R}^{n \times n}$ is defined as
   \[ \|A\| \equiv \max_{\|x\|=1} \|Ax\|, \]
   where $x \in \mathbb{R}^n$, $\|x\|$ denotes the vector norm of $x$, and $Ax$ denotes matrix-vector multiplication.
   (a) Prove that if $I$ is the $2 \times 2$ identity matrix, i.e.
       \[ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \]
       then $\|I\| = 1$.
   (b) Prove that the norm of the matrix
       \[ A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \]
       is 2.
   (c) Extend your argument in the last problem to argue that the norm of any real valued diagonal matrix is just the maximum of the absolute value of the entries on the diagonal.
   (d) Finally, argue that the norm of the inverse of any diagonal matrix (with non-zero entries on the diagonal) is just the reciprocal of the absolute value of the smallest element on the diagonal.
6. (Practice with Hermitian matrices.) Let $A \in \mathbb{R}^{n \times n}$ be a matrix, with eigenvectors $u_1, \ldots, u_n$ in $\mathbb{R}^n$ and corresponding eigenvalues $\lambda_1, \ldots, \lambda_n$. If the $u_i$ are orthogonal to one another, they are said to comprise an orthogonal eigenbasis for $A$, and $A$ can be written with respect to this basis as a diagonal matrix with diagonal entries $\lambda_1, \ldots, \lambda_n$. A fundamental theorem in linear algebra is that any real valued Hermitian matrix has an orthogonal eigenbasis.
   (a) Show that in general, eigenvectors need not be orthogonal to one another. Do this by finding the eigenvectors of the matrix
       \[ A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \]
       and verifying that their dot product is not zero.
   (b) Convince yourself that the any real valued Hermitian matrix really does have orthogonal eigenvectors by inventing a $2 \times 2$ Hermitian matrix, finding its eigenvectors, and verifying that they are orthogonal.
   (c) Use the fact that Hermitian matrices have an orthogonal eigenbasis, together with your work in Problem 5., to show that the norm of the Hessian matrix is given by the absolute value of its largest eigenvalue, and the norm of its inverse by the reciprocal of the absolute value of its smallest eigenvalue.
   (d) Finally, use the fact that Hermitian matrices have an orthogonal eigenbasis to show that the Hessian is positive definite if and only if its eigenvalues are positive.