Homework 8
Due Monday, November 4

Individual Problems:

1. (7.4.13) Two samples, each of size \(n\), are taken from a normal distribution with unknown mean \(\mu\) and unknown standard deviation \(\sigma\). A 90% confidence interval for \(\mu\) is constructed with the first sample, and a 95% confidence interval for \(\mu\) is constructed with the second. Will the 95% confidence interval necessarily be longer than the 90% interval? Explain.

2. (7.2.24) Suppose one hundred samples of size \(n = 3\) are taken from each of the pdfs
\[f_y(y) = 2y, \quad 0 \leq y \leq 1\]
and
\[f_Y(y) = 4y^3, \quad 0 \leq y \leq 1.\]
For each set of three observations, the ratio
\[\frac{\bar{y} - \mu}{s/\sqrt{3}}\]
where \(\mu\) is the mean of the pdf being sampled. How would you expect the distributions of the two sets of ratios to be different? How would they be similar? Be as specific as possible.

3. (7.5.8) A random sample of size \(n = 19\) is drawn from a normal distribution for which \(\sigma^2 = 12.0\). In what range are we likely to find the sample variance, \(s^2\)? Answer the question by finding two numbers \(a\) and \(b\) such that
\[P(a \leq S^2 \leq b) = 0.95.\]

Group Problems

1. (7.3.4) Use the fact that \((n-1)S^2/\sigma^2\) is a chi square random variable with \(n-1\) df to prove that
\[\text{Var}(S^2) = \frac{2\sigma^4}{n-1}.\]
(Hint: Use the fact that the variance of a chi square random variable with \(k\) df is \(2k\).)

2. (7.4.5) Let \(\bar{Y}\) and \(S\) denote the sample and sample standard deviation, respectively, based on a set of \(n = 20\) measurements taken from a normal distribution with \(\mu = 90.6\) Find the function \(k(S)\) for which
\[P[90.6 - k(S) \leq \bar{Y} \leq 90.6 + k(S)] = 0.99.\]

3. (7.5.6) Let \(Y_1, \ldots, Y_n\) be a random sample of size \(n\) from a normal distribution having mean \(\mu\) and variance \(\sigma^2\). What is the smallest value of \(n\) for which the following is true?
\[P\left(\frac{S^2}{\sigma^2} < 2\right) \geq 0.95.\]
(Hint: use a trial and error method.)